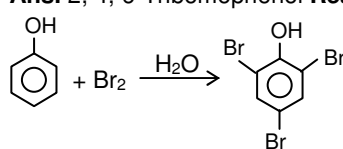


<b>CHEMSITRY</b>											
1.	<p>Silver Mirror test is given by which one of the following compounds ? (a) Formaldehyde (b) Benzophenone (c) Acetaldehyde (d) Acetone</p> <p><b>Ans.</b> Formaldehyde &amp; Acetaldehyde <b>Reason:</b> Both HCHO &amp; CH<sub>3</sub>CHO responds Tollen's reagent.</p>										
2.	<p>A 5.2 molal aqueous solution of methyl alcohol, CH<sub>3</sub>OH, is supplied. What is the mole fraction of methyl alcohol in the solution ? (a) 0.086 (b) 0.050 (c) 0.100 (d) 0.190</p> <p><b>Ans.</b> 0.086 <b>Reason:</b> <math>m.f = \frac{5.2}{5.2 + \frac{1000}{18}}</math></p>										
3.	<p>Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is (a) 2, 2, 2-Trichloropropanol (b) Chloroform (c) 2, 2, 2-Trichloroethanol (d) Trichloromethanol.</p> <p><b>Ans.</b> 2, 2, 2-Trichloroethanol <b>Reason:</b> <math>CCl_3CHO \xrightarrow{NaOH} CCl_3COONa + CCl_3CH_2OH</math>.</p>										
4.	<p>The rate of a chemical reaction doubles for every 10°C rise of temperature. If the temperature is raised by 50°C, the rate of the reaction increases by about (a) 32 times (b) 64 times (c) 10 times (d) 24 times.</p> <p><b>Ans.</b> 32 times. <b>Reason:</b> Rate of reaction increases by 2<sup>5</sup> times.</p>										
5.	<p>'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because (a) a for Cl<sub>2</sub> &lt; a for C<sub>2</sub>H<sub>6</sub> but b for Cl<sub>2</sub> &gt; b for C<sub>2</sub>H<sub>6</sub> (b) a for Cl<sub>2</sub> &gt; a for C<sub>2</sub>H<sub>6</sub> but b for Cl<sub>2</sub> &lt; b for C<sub>2</sub>H<sub>6</sub> (c) a and b for Cl<sub>2</sub> &gt; a and b for C<sub>2</sub>H<sub>6</sub> (d) a and b for Cl<sub>2</sub> &lt; a and b for C<sub>2</sub>H<sub>6</sub></p> <p><b>Ans.</b> a for Cl<sub>2</sub> &gt; a for C<sub>2</sub>H<sub>6</sub> but b for Cl<sub>2</sub> &lt; b for C<sub>2</sub>H<sub>6</sub></p> <p><b>Reason:</b></p> <table style="margin-left: 20px;"> <thead> <tr> <th>Substance</th> <th>a</th> <th>b</th> </tr> </thead> <tbody> <tr> <td>C<sub>2</sub>H<sub>6</sub></td> <td>5.562</td> <td>0.0638</td> </tr> <tr> <td>Cl<sub>2</sub></td> <td>6.43</td> <td>0.0542</td> </tr> </tbody> </table>	Substance	a	b	C <sub>2</sub> H <sub>6</sub>	5.562	0.0638	Cl <sub>2</sub>	6.43	0.0542	
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C <sub>2</sub> H <sub>6</sub>	5.562	0.0638									
Cl <sub>2</sub>	6.43	0.0542									
6.	<p>The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm<sup>3</sup> to a volume of 100 dm<sup>3</sup> at 27°C is (a) 32.3 J mol<sup>-1</sup> K<sup>-1</sup> (b) 42.3 J mol<sup>-1</sup> K<sup>-1</sup> (c) 38.3 J mol<sup>-1</sup> K<sup>-1</sup> (d) 35.8 J mol<sup>-1</sup> K<sup>-1</sup>.</p> <p><b>Ans.</b> 38.3 J mol<sup>-1</sup> K<sup>-1</sup> <b>Reason:</b> <math>\Delta s = 2.303 nR \log \frac{V_2}{V_1}</math></p> <p><math>= 2.303 \times 2 \times 8,314 \times \log \frac{100}{10} = 38.3 \text{ J mol}^{-1} \text{ K}^{-1}</math>.</p>										
7.	<p>A vessel at 1000 K contains CO<sub>2</sub> with a pressure of 0.5 atm. Some of the CO<sub>2</sub> is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is (a) 0.3 atm (b) 0.18 atm (c) 1.8 atm (d) 3 atm.</p> <p><b>Ans.</b> 1.8 atm <b>Reason:</b> <math>CO_2 + C \rightleftharpoons 2CO</math></p> <table style="margin-left: 20px;"> <tbody> <tr> <td style="text-align: center;">0.5</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="text-align: center;">0.5 - x</td> <td style="text-align: center;">2x</td> </tr> <tr> <td colspan="2" style="text-align: center;">0.5 - x + 2x = 0.8</td> </tr> <tr> <td colspan="2" style="text-align: center;">x = 0.3</td> </tr> <tr> <td colspan="2" style="text-align: center;"><math>K_p = \frac{(0.6)^2}{0.2} = 1.8 \text{ atm}</math></td> </tr> </tbody> </table>	0.5	0	0.5 - x	2x	0.5 - x + 2x = 0.8		x = 0.3		$K_p = \frac{(0.6)^2}{0.2} = 1.8 \text{ atm}$	
0.5	0										
0.5 - x	2x										
0.5 - x + 2x = 0.8											
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$K_p = \frac{(0.6)^2}{0.2} = 1.8 \text{ atm}$											
8.	<p>A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at (a) 743 nm (b) 518 nm (c) 1035 nm (d) 325 nm.</p> <p><b>Ans.</b> 743 nm <b>Reason:</b> <math>\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}</math></p>										

	$\Rightarrow \frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}, \lambda_2 = 743 \text{ nm.}$
9.	<p>In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is (a) <math>A_2B_3</math> (b) <math>A_2B_5</math> (c) <math>A_2B</math> (d) <math>AB_2</math></p> <p><b>Ans.</b> <math>A_2B_5</math> <b>Reason:</b> Number of A atoms = <math>8 \times \frac{1}{8} = 1</math></p> <p>B atoms = <math>5 \times \frac{1}{2} = \frac{5}{2}</math>, formula is <math>A_2B_5</math></p>
10.	<p>Among the following the maximum covalent character is shown by the compound (a) <math>AlCl_3</math> (b) <math>MgCl_2</math> (c) <math>FeCl_2</math> (d) <math>SnCl_2</math>.</p> <p><b>Ans.</b> <math>AlCl_3</math> <b>Reason:</b> .</p>
11.	<p>Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides ? (a) <math>Na_2O &lt; K_2O &lt; MgO &lt; Al_2O_3</math> (b) <math>K_2O &lt; Na_2O &lt; Al_2O_3 &lt; MgO</math> (c) <math>Al_2O_3 &lt; MgO &lt; Na_2O &lt; K_2O</math>. (d) <math>MgO &lt; K_2O &lt; Al_2O_3 &lt; Na_2O</math>.</p> <p><b>Ans.</b> <math>Al_2O_3 &lt; MgO &lt; Na_2O &lt; K_2O</math>.</p>
12.	<p>The structure of <math>IF_7</math> is (a) octahedral (b) pentagonal bipyramid (c) square pyramid (d) trigonal bipyramid.</p> <p><b>Ans.</b> pentagonal bipyramid</p>
13.	<p>Identify the compound that exhibits tautomerism (a) 2-Pentanone (b) Phenol (c) 2-Butene (d) Lactic acid.</p> <p><b>Ans.</b> 2-Pentanone <b>Reason:</b> <math>CH_3-\overset{\overset{O}{  }}{C}-CH_2-CH_2-CH_3</math></p>
14.	<p>Phenol is heated with a solution of mixture of <math>KBr</math> and <math>KBrO_3</math>. The major product obtained in the above reaction is (a) 4-Bromophenol (b) 2, 4, 6-Tribromophenol (c) 2-Bromophenol (d) 3-Bromophenol</p> <p><b>Ans.</b> 2, 4, 6-Tribromophenol <b>Reason:</b> <math>5 Br^- + BrO_3^- + 6 H^+ \longrightarrow 3 Br_2 + 3 H_2O</math></p> 
15.	<p>Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at <math>-6^\circ C</math> will be : (<math>K_f</math> for water = <math>1.86 \text{ K kg mol}^{-1}</math>, and molar mass of ethylene glycol = <math>62 \text{ mol}^{-1}</math>) (a) 400.00 g (b) 304.60 g (c) 804.32 g (d) 204.30 gm.</p> <p><b>Ans.</b> 804.32 g <b>Reason:</b> <math>\Delta T_f = K_f \times \frac{W}{M} \times \frac{1000}{W}</math></p> <p><math>6 = 1.86 \times \frac{W}{62} \times \frac{1000}{4000}</math>, <math>W = 800 \text{ gm.}</math></p>
16.	<p>The degree of dissociation (<math>\alpha</math>) of a weak electrolyte, <math>A_xB_y</math> is related to van't Hoff factor (<math>i</math>) by the expression : (a) <math>\alpha = \frac{x+y-1}{i-1}</math> (b) <math>\alpha = \frac{x+y+1}{i-1}</math> (c) <math>\alpha = \frac{i-1}{(x+y-1)}</math> (d) <math>\alpha = \frac{i-1}{x+y+1}</math>.</p>

	<b>Ans.</b> $\alpha = \frac{i-1}{(x+y-1)}$
17.	Boron cannot form which one of the following anions ? (a) $\text{B}(\text{OH})_4^-$ (b) $\text{BO}_2^-$ (c) $\text{BF}_6^{3-}$ (d) $\text{BH}_4^-$ . <b>Ans.</b> $\text{BF}_6^{3-}$
18.	The strongest acid amongst the following compounds is : (a) $\text{CH}_3\text{CH}_2\text{CH}(\text{Cl})\text{CO}_2\text{H}$ (b) $\text{ClCH}_2\text{CH}_2\text{CH}_2\text{COOH}$ (c) $\text{CH}_3\text{COOH}$ (d) $\text{HCOOH}$ . <b>Ans.</b> $\text{CH}_3\text{CH}_2\text{CH}(\text{Cl})\text{CO}_2\text{H}$
19.	Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is (a) Ethyl chloride (b) Ethyl ethanoate (c) Diethyl ether (d) 2 – Butanone. $\begin{array}{c} \text{O} \\    \\ \text{CH}_3 - \text{C} - \text{Cl} + \text{NaOC}_2\text{H}_5 \end{array} \longrightarrow \begin{array}{c} \text{O} \\    \\ \text{CH}_3 - \text{C} - \text{OC}_2\text{H}_5 \end{array}$ <b>Ans.</b> Ethyl ethanoate <b>Reason :</b>
20.	The presence or absence of hydroxyl group on which carbon atom of sugar differentiates RNA and DNA ? (a) 3 <sup>rd</sup> (b) 4 <sup>th</sup> (c) 1 <sup>st</sup> (d) 2 <sup>nd</sup> . <b>Ans.</b> 2 <sup>nd</sup> .
21.	The outer electron configuration of Gd (Atomic No : 64) is : (a) $4f^4 5d^4 6s^2$ (b) $4f^7 5d^7 6s^2$ (c) $4f^3 5d^5 6s^2$ (d) $4f^8 5d^0 6s^2$ . <b>Ans.</b> $4f^7 5d^1 6s^2$
22.	The magnetic moment (spin only) of $[\text{NiCl}_4]^{2-}$ is : (a) 2.82 BM (b) 1.41 BM (c) 1.82 BM (d) 5.46 BM. <b>Ans.</b> 2.82 BM. <b>Reason :</b> No. of unpaired e <sup>-</sup> = 2 $\mu = \sqrt{n(n+2)} = \sqrt{8} = 2.82 \text{ BM}$ .
23.	The hybridization of orbitals of N atom in $\text{NO}_3^-$ , $\text{NO}_2^+$ and $\text{NH}_4^+$ are respectively : (a) sp, $\text{sp}^3$ , $\text{sp}^2$ (b) $\text{sp}^2$ , $\text{sp}^3$ , sp (c) sp, $\text{sp}^2$ , $\text{sp}^3$ (d) $\text{sp}^2$ , sp, $\text{sp}^3$ . <b>Ans.</b> $\text{sp}^2$ , sp, $\text{sp}^3$ . <b>Reason :</b> $\text{NO}_3^- \rightarrow \text{sp}^2$ , $\text{NO}_2^+ \rightarrow \text{sp}$ , $\text{NH}_4^+ \rightarrow \text{sp}^3$ .
24.	Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of (a) an isopropyl group (b) an acetylenic triple bond (c) two ethylenic double bonds (d) a vinyl group. <b>Ans.</b> a vinyl group <b>Reason :</b> $\text{R} - \text{CH} = \text{CH}_2 \xrightarrow[\text{Zn}/\text{H}_2\text{O}]{\text{O}_3/\text{CCl}_4} \text{R} - \text{CHO} + \text{HCHO}$ .
25.	Which of the following facts about the complex $[\text{Cr}(\text{NH}_3)_6]\text{Cl}_3$ is wrong ? (a) The complex is an outer orbital complex (b) The complex gives white precipitate with silver nitrate solution (c) The complex involves $d^2\text{sp}^3$ hybridisation and is octahedral in shape (d) The complex is paramagnetic. <b>Ans.</b> The complex is an outer orbital complex.
26.	The reduction potential of hydrogen half-cell will be negative if : (a) $p(\text{H}_2) = 2 \text{ atm}$ and $[\text{H}^+] = 1.0 \text{ M}$ (b) $p(\text{H}_2) = 2 \text{ atm}$ and $[\text{H}^+] = 2.0 \text{ M}$ (c) $p(\text{H}_2) = 1 \text{ atm}$ and $[\text{H}^+] = 2.0 \text{ M}$ (d) $p(\text{H}_2) = 1 \text{ atm}$ and $[\text{H}^+] = 1.0 \text{ M}$ . <b>Ans.</b> $p(\text{H}_2) = 2 \text{ atm}$ and $[\text{H}^+] = 1.0 \text{ M}$ . <b>Reason.</b> $E = E^\circ - \frac{0.059}{n} \log \frac{p_{\text{H}_2}}{[\text{H}^+]^2}$ .
27.	In context of the lanthanoids, which of the following statements is not correct ? (a) Because of similar properties the separation of lanthanoids is not easy (b) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series (c) There is a gradual decrease in the radii of the members with increasing atomic number in the series (d) All the members exhibit +3 oxidation state. <b>Ans.</b> Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.
28.	Which of the following reagents may be used to distinguish between phenol and benzoic acid ? (a) Molisch reagent (b) Neutral $\text{FeCl}_3$ (c) Aqueous NaOH (d) Tollen's reagent. <b>Ans.</b> Neutral $\text{FeCl}_3$ <b>Reason .</b> Phenol gives +ve test with Neutral $\text{FeCl}_3$ .
29.	Which of the following statements regarding sulphur is incorrect ? (a) At $600^\circ\text{C}$ the gas mainly consists of $\text{S}_2$ molecules (b) The oxidation state of sulphur is never less than +4 in its compound (c)

	<p><math>S_2</math> molecule is paramagnetic (d) The vapour at <math>200^\circ\text{C}</math> consists mostly of <math>S_8</math> rings.  <b>Ans.</b> The oxidation state of sulphur is never less than +4 in its compound. <b>Reason.</b> Sulphur shows oxidation state of -2 to +6.</p>
30.	<p>Which of the following statement is wrong ? (a) Single N – N bond is weaker than the single P – P bond (b) <math>N_2O_4</math> has two resonance structures (c) The stability of hydrides increases from <math>NH_3</math> to <math>BiH_3</math> in group 15 of the periodic table (d) Nitrogen cannot form <math>d\pi - p\pi</math> bond.  <b>Ans.</b> The stability of hydrides increases from <math>NH_3</math> to <math>BiH_3</math> in group 15 of the periodic table. <b>Reason.</b> Stability of hydrides decreases from <math>NH_3</math> to <math>BiH_3</math>.</p>

### MATHEMATICS

31.	<p>If <math>\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})\sqrt{a^2 + b^2}</math> and <math>\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})</math>, then the value of <math>\left( (2\vec{a} - \vec{b}) \cdot \left[ (\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b}) \right] \right)</math> is: (a) 5 (b) 3 (c) -5 (d) -3.  <b>Ans.</b> -5 <b>Reason.</b> <math>(2\vec{a} - \vec{b}) \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]</math>  <math>\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})</math>  <math>\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})</math>  <math>\hat{a} \times \hat{b}</math> is an unit vector  <math>= [(\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b})] \cdot (\vec{a} \times \vec{b})</math>  <math>= -5  \vec{a} \times \vec{b} ^2</math>  <math>= -5(1)^2</math>  <math>= -5</math></p>															
32.	<p>The coefficient of <math>x^7</math> in the expansion of <math>(1 - x - x^2 + x^3)^6</math> is: (a) -144 (b) 132 (c) 144 (d) -132.  <b>Ans.</b> -144 <b>Reason.</b> <math>(1 - x - x^2 + x^3)^6</math>  <math>(1 - x)^6 (1 - x^2)^6</math>  <math>= \sum_{r=0}^6 {}^6C_r (-x)^r \sum_{s=0}^6 {}^6C_s (-x^2)^s</math>          Coeff of <math>x^7</math>  <math>r + 2s = 7</math></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>r</td> <td>s</td> <td></td> </tr> <tr> <td>7</td> <td>0</td> <td>x</td> </tr> <tr> <td>5</td> <td>1</td> <td></td> </tr> <tr> <td>3</td> <td>2</td> <td></td> </tr> <tr> <td>1</td> <td>3</td> <td></td> </tr> </table> <p><math>{}^6C_5 (-1)^5 \cdot {}^6C_1 (-1)^1 + {}^6C_3 (-1)^3 \cdot {}^6C_2 (-1)^2 + {}^6C_1 (-1)^1 \cdot {}^6C_3 (-1)^3</math>  <math>= 36 - 300 + 120</math>  <math>= 156 - 300</math>  <math>= -144</math></p>	r	s		7	0	x	5	1		3	2		1	3	
r	s															
7	0	x														
5	1															
3	2															
1	3															
33.	<p>Let <math>\alpha, \beta</math> be real and <math>z</math> be a complex number. If <math>z^2 + \alpha z + \beta = 0</math> has two distinct roots on the line <math>\text{Re } z</math></p>															

	<p>= 1, then it is necessary that : (a) <math> \beta  = 1</math> (b) <math>\beta \in (1, \infty)</math> (c) <math>\beta \in (0, 1)</math> (d) <math>\beta \in (-1, 0)</math>.</p> <p><b>Ans.</b> <math>\beta \in (1, \infty)</math> <b>Reason.</b> <math>\alpha, \beta \in \mathbb{R}</math></p> <p><math>z^2 + \alpha z + \beta = 0</math> has 2 distinct roots on <math>\text{Re}(Z) = 1</math></p> <p>Let <math>z = 1 + iy</math></p> <p><math>z^2 = 1 - y^2 + 2iy</math></p> <p>Now <math>(1 - y^2 + 2iy) + \alpha(1+iy) + \beta = 0</math></p> <p><math>(1 - y^2 + \alpha + \beta) + i(iy + \alpha y) = 0 + i.0</math></p> <p><math>2y + \alpha y = 0</math></p> <p><math>\Rightarrow y = 0</math> or <math>\alpha = -z</math></p> <p><math>1 - y^2 + \alpha + \beta = 0</math></p> <p><math>1 - 2 + \beta + y^2 = 0</math></p> <p><math>y^2 = \beta - 1</math></p> <p><math>y = \pm\sqrt{\beta-1}</math></p> <p><math>\beta - 1 &gt; 0</math></p> <p><math>\beta &gt; 1</math></p> <p><math>\beta \in (1, \infty)</math></p> <p>option (b).</p>
34.	<p>Consider the following statements</p> <p>P: Suman is brilliant</p> <p>Q: Suman is rich</p> <p>S: Suma is honest</p> <p>The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as:</p> <p>(a) <math>\sim Q \leftrightarrow \sim P \wedge R</math> (b) <math>\sim (P \wedge \sim R) \leftrightarrow O</math> (c) <math>\sim P \wedge (Q \leftrightarrow \sim R)</math> (d) <math>\sim (O \leftrightarrow (P \wedge \sim R))</math>.</p> <p><b>Ans.</b> <math>\sim (O \leftrightarrow (P \wedge \sim R))</math> <b>Reason.</b> Suman is brilliant and dishonest <math>\leftrightarrow</math> Suman is rich</p> <p><math>\Rightarrow (P \wedge \sim R) \leftrightarrow Q</math></p> <p><math>\Rightarrow Q \leftrightarrow (P \wedge \sim R)</math></p> <p>Its negation is <math>\sim(Q \leftrightarrow (P \wedge \sim R))</math></p>
35.	<p><math>\frac{d^2x}{dy^2}</math> equals (a) <math>\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}</math> (b) <math>-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}</math> (c) <math>\left(\frac{d^2y}{dx^2}\right)^{-1}</math> (d) <math>-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}</math>.</p> <p><b>Ans.</b> <math>-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}</math> <b>Reason.</b> <math>\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{dy}{dx}\right)^{-1}</math></p> <p><math>= (-1)\left(\frac{dy}{dx}\right)^{-2} \frac{d}{dy}\left(\frac{dy}{dx}\right)</math></p> <p><math>= (-1)\left(\frac{dy}{dx}\right)^{-2} \frac{d}{dx}\left(\frac{dy}{dx}\right) \cdot \frac{dx}{dy}</math></p> <p><math>= (-1)\left(\frac{dy}{dx}\right)^{-2} \frac{d^2y}{dx^2} \frac{1}{\left(\frac{dy}{dx}\right)}</math></p> <p><math>= (-1)\left(\frac{dy}{dx}\right)^{-3} \frac{d^2y}{dx^2}</math></p>
36.	<p>Statement – 1:</p> <p>The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line: <math>\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}</math>.</p>

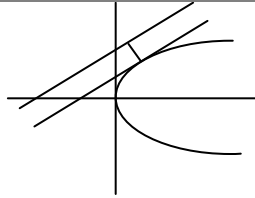
	<p>Statement – 2:  The line: <math>\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}</math> bisects the line segment joining A (1, 0, 7) and B(1, 6, 3)</p> <p>(a) Statement – 1 is true, Statement – 2 is false  (b) Statement – 1 is false, Statement – 2 is true.  (c) Statement – 1 is true, Statement – 2 is true; Statement – 2 is correct explanation for Statement – 1  (d) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1.</p> <p><b>Ans.</b> Statement – 1 is true, Statement – 2 is true; Statement – 2 is correct explanation for Statement – 1 <b>Reason.</b> A (1, 0, 7), B(1, 6, 3)  Mid point of AB is C(1, 3, 5)  Clearly C(1, 3, 5) satisfies the line <math>\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}</math></p>
<p>37.</p>	<p>If C and D are two events such that <math>C \subset D</math> and <math>P(D) \neq 0</math>, then the correct statement among the following is (a) <math>P(C/D) &lt; P(C)</math> (b) <math>P(C/D) &lt; \frac{P(D)}{P(C)}</math> (c) <math>P(C/D) = P(C)</math> (d) <math>P(C/D) &gt; P(C)</math>.</p> <p><b>Ans.</b> <math>P(C/D) &lt; P(C)</math> <b>Reason:</b> Given <math>C \subset D</math>  <math>\Rightarrow P(C) &lt; P(D)</math>, <math>P(D) &gt; 0</math>  Now <math>C \cap D = C</math>  <math>\Rightarrow P(C \cap D) &lt; P(C)</math>  <math>\Rightarrow P(C \cap D) &lt; P(C) \cdot P(D)</math>  <math>\Rightarrow \frac{P(C \cap D)}{P(D)} &lt; P(C) \Rightarrow P(C/D) &lt; P(C)</math></p>
<p>38.</p>	<p>Consider 5 independent Bernoulli's trials each with probability of success p. if the probability of at least one failure is greater than or equal to <math>\frac{31}{32}</math>, then p lies in the interval: (a) <math>[0, 1/2]</math> (b) <math>[\frac{11}{12}, 1]</math>  (c) <math>[\frac{1}{2}, \frac{3}{4}]</math> (d) <math>[\frac{3}{4}, \frac{11}{12}]</math>.</p> <p><b>Ans.</b> <math>[0, 1/2]</math> <b>Reason.</b> 5 trials  p = success  q = 1 – p = failure  P (Atleast one failure)  <math>\Rightarrow 1 - P</math> (all success)  <math>\Rightarrow 1 - p^5 \geq \frac{31}{32}</math>  <math>\Rightarrow p^5 \leq \frac{1}{32}</math>  <math>\Rightarrow p^2 \leq \frac{1}{32}</math>  <math>\Rightarrow p \leq \frac{1}{2}</math>  as <math>P \geq 0</math>  So <math>0 \leq p \leq \frac{1}{2}</math>  <math>P \in [0, \frac{1}{2}]</math>.</p>
<p>39.</p>	<p>The value of <math>\int_0^1 \frac{8 \log(1+x)}{1+x^2} dx</math> is: (a) <math>\frac{\pi}{2} \log 2</math> (b) <math>\log 2</math> (c) <math>\pi \log 2</math> (d) <math>\frac{\pi}{8} \log 2</math>.</p>

	<p><b>Ans.</b> <math>\pi \log 2</math> <b>Reason.</b> <math>\int_0^1 \frac{8 \log(1+x)}{1+x^2} dx = 8 \int_0^{\pi/4} \frac{\log(1+\tan \theta) \sec^2 \theta d\theta}{\sec^2 \theta}</math></p> <p>Let <math>x = \tan \theta</math>  <math>dx = \sec^2 \theta d\theta</math>  <math>x \rightarrow 0, \theta \rightarrow 0</math>  <math>x \rightarrow 1, \theta \rightarrow \pi/4</math></p> <p>Now <math>\int_0^{\pi/4} \log(1+\tan \theta) d\theta = \int_0^{\pi/4} \log(1+\tan(\pi/4 - \theta)) d\theta</math></p> <p><math>= \int_0^{\pi/4} \log 2 d\theta - \int_0^{\pi/4} \log(1+\tan \theta) d\theta</math></p> <p><math>\Rightarrow 2 \int_0^{\pi/4} \log(1+\tan \theta) d\theta = \log 2 \cdot \pi/4</math></p> <p><math>\Rightarrow \int_0^{\pi/4} \log(1+\tan \theta) d\theta = \frac{\pi}{8} \log 2</math></p> <p><math>= 8 \int_0^{\pi/4} \log(1+\tan \theta) d\theta</math></p> <p><math>= 8 \cdot \frac{\pi}{8} \log 2</math></p> <p><math>= \pi \cdot \log 2.</math></p>
40.	<p>For <math>x \in \left(0, \frac{5\pi}{2}\right)</math>, define <math>f(x) = \int_0^x \sqrt{t} \sin t dt</math>, then it was (a) local minimum at <math>\pi</math> and local maximum at <math>2\pi</math> (b) local maximum at <math>\pi</math> and local minimum at <math>2\pi</math> (c) local maximum at <math>\pi</math> and <math>2\pi</math> (d) local minimum at <math>\pi</math> and <math>2\pi</math>.</p> <p><b>Ans.</b> local maximum at <math>\pi</math> and local minimum at <math>2\pi</math> <b>Reason.</b> <math>x \in (0, 5\pi/2)</math></p> <p><math>f(x) = \int_0^x \sqrt{t} \sin t dt</math></p> <p><math>f'(x) = \sqrt{x} \sin x</math></p> <p><math>f''(x) = \sqrt{x} \cos x + \frac{\sin x}{2\sqrt{x}}</math></p> <p><math>f'(x) = 0 \Rightarrow x = 0, n\pi, n \in \mathbb{Z}</math></p> <p><math>x = 0, \pi, 2\pi</math></p> <p><math>f''(\pi) = -\sqrt{\pi} &lt; 0</math> (maximum)</p> <p><math>f''(2\pi) = \sqrt{2\pi} &gt; 0</math> (minimum)</p>
41.	<p>The vectors <math>\vec{a}</math> and <math>\vec{b}</math> are not perpendicular <math>\vec{c}</math> and <math>\vec{d}</math> are two vectors satisfying: <math>\vec{b} \times \vec{c} = \vec{b} \times \vec{d}</math> and <math>\vec{a} \cdot \vec{d} = 0</math>, then the vector <math>\vec{d}</math> is equal to: (a) <math>\vec{b} + \frac{\begin{pmatrix} \vec{a} \cdot \vec{c} \\ \vec{a} \cdot \vec{b} \end{pmatrix}}{\vec{a} \cdot \vec{b}} \vec{c}</math> (b) <math>\vec{c} - \frac{\begin{pmatrix} \vec{a} \cdot \vec{c} \\ \vec{a} \cdot \vec{b} \end{pmatrix}}{\vec{a} \cdot \vec{b}} \vec{b}</math> (c) <math>\vec{b} - \frac{\begin{pmatrix} \vec{a} \cdot \vec{c} \\ \vec{a} \cdot \vec{b} \end{pmatrix}}{\vec{a} \cdot \vec{b}} \vec{c}</math> (d) <math>\vec{c} + \frac{\begin{pmatrix} \vec{a} \cdot \vec{c} \\ \vec{a} \cdot \vec{b} \end{pmatrix}}{\vec{a} \cdot \vec{b}} \vec{b}</math></p> <p><b>Ans.</b> <math>\vec{c} - \frac{\begin{pmatrix} \vec{a} \cdot \vec{c} \\ \vec{a} \cdot \vec{b} \end{pmatrix}}{\vec{a} \cdot \vec{b}} \vec{b}</math> <b>Reason.</b> <math>\vec{a} \perp \vec{d} \Rightarrow \vec{a} \cdot \vec{b} \neq 0, \vec{a} \cdot \vec{d} = 0</math></p> <p><math>\vec{b} \times \vec{c} = \vec{b} \times \vec{d}</math></p> <p>Using cross product with <math>\vec{a}</math></p>

	$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{d})$ $\Rightarrow (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c} = (\vec{a} \cdot \vec{d}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{d}$ $\Rightarrow (\vec{a} \cdot \vec{b}) \vec{d} = (\vec{a} \cdot \vec{b}) \vec{c} - (\vec{a} \cdot \vec{c}) \vec{b}$ <p>Dividing <math>(\vec{a} \cdot \vec{b})</math></p> $\Rightarrow \vec{d} = \vec{c} - \left( \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \right) \vec{b}$
42.	<p>Let <math>R</math> be the set of real numbers.</p> <p><b>Statement – 1:</b> <math>A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}</math> an equivalence relation on <math>R</math>.</p> <p><b>Statement – 2:</b> <math>B = \{(x, y) \in R \times R : x = \alpha y \text{ for some rational number } \alpha\}</math> is an equivalence relation on <math>R</math>.</p> <p>(a) Statement – 1 is true, Statement – 2 is false          (b) Statement – 1 is false, Statement – 2 is true.          (c) Statement – 1 is true, Statement – 2 is true; Statement – 2 is correct explanation for Statement – 1          (d) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1.</p> <p><b>Ans.</b> statement – 1 is true, statement – 2 is true; statement – 2 is not a correct explanation for statement – 1. <b>Reason.</b> <math>A = \{(x, y) / (y - x) \text{ is an integer}\}</math> in <math>R</math>  <math>A</math> is Reflexive : <math>(x - x)</math> is an integer.  <math>A</math> is Symmetric : <math>y - x \in Z \Rightarrow x - y \in Z</math>  <math>A</math> is Transitive : <math>x - y \in Z, y - z \in Z \Rightarrow x - y + y - z = x - z \in Z</math>  <math>B = \{(x, y) / x = \alpha y\}</math> in <math>R</math>  <math>B</math> is Reflexive : <math>x = 1x, 1 \in Q</math>  <math>B</math> is Symmetric : <math>(x, y) \in R \Rightarrow x = \alpha y \Rightarrow y = \frac{1}{\alpha} x \Rightarrow (y, x) \in R, \frac{1}{\alpha} \in Q</math>  <math>B</math> is Transitive : <math>(x, y) \in R, (y, z) \in R \Rightarrow x = \alpha Y, y = \beta Z</math>  <math>\Rightarrow x = \alpha\beta Z \Rightarrow (x, z) \in R</math>.          Hence, <math>A</math> and <math>B</math> are both equivalence relations.</p>
43.	<p>Let <math>A</math> and <math>B</math> be two symmetric matrices of order 3.</p> <p><b>Statement – 1:</b>  <math>A(BA)</math> and <math>(AB)A</math> are symmetric matrices</p> <p><b>Statement – 2:</b>  <math>AB</math> is symmetric matrix of matrix multiplication of <math>A</math> with <math>B</math> is commutative.</p> <p>(a) Statement – 1 is true, Statement – 2 is false          (b) Statement – 1 is false, Statement – 2 is true.          (c) Statement – 1 is true, Statement – 2 is true; Statement – 2 is correct explanation for Statement – 1          (d) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1.</p> <p><b>Ans.</b> Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1. <b>Reason.</b> Given <math>A</math> is symmetric <math>\Rightarrow A^T = A</math>  <math>B</math> is symmetric <math>\Rightarrow B^T = B</math>          Now <math>((AB)A)^T = A^T(AB)^T = A^T(B^T A^T) = A(BA) = (AB)A</math>          Hence <math>(AB)A</math> is symmetric.          Similarly <math>A(BA)</math> is also symmetric.          Now <math>(AB)^T = B^T A^T = BA = AB</math> (If <math>A</math> and <math>B</math> obey the commutative law of mult. Otherwise <math>AB</math> is not symmetric)</p>

44.	<p>The two circles <math>x^2 + y^2 = ax</math> and <math>x^2 + y^2 = c^2</math> (<math>c &gt; 0</math>) touch each other if: (a) <math>a = 2c</math> (b) <math> a  = 2c</math> (c) <math>2 a  = c</math> (d) <math> a  = c</math>.</p> <p><b>Ans.</b> <math> a  = c</math>. <b>Reason.</b> <math>x^2 + y^2 = ax \rightarrow</math> Center <math>\left(\frac{a}{2}, 0\right)</math>, <math>r_1 = \frac{a}{2}</math></p> <p><math>x^2 + y^2 = c^2 \xrightarrow{(c&gt;0)}</math> center <math>(0, 0)</math>, <math>r_2 = c</math></p> <p>Touch each other if Distance between the centers = <math> r_1 \pm r_2 </math></p> <table style="border: none;"> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"><math>\frac{a}{2} = \left  \frac{a}{2} \pm c \right </math></td> <td><math>c = a, -a</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"><math>\left  c \mp \frac{a}{2} \right  = \frac{a}{2}</math></td> <td>i.e. <math>c =  a </math>.</td> </tr> </table> <p><math>c \mp \frac{a}{2} = \pm \frac{a}{2}</math></p> <p><math>c = \pm \frac{a}{2} \pm \frac{a}{2}</math></p> <p><math>c = a, 0, -a</math></p>	$\frac{a}{2} = \left  \frac{a}{2} \pm c \right $	$c = a, -a$	$\left  c \mp \frac{a}{2} \right  = \frac{a}{2}$	i.e. $c =  a $ .
$\frac{a}{2} = \left  \frac{a}{2} \pm c \right $	$c = a, -a$				
$\left  c \mp \frac{a}{2} \right  = \frac{a}{2}$	i.e. $c =  a $ .				
45.	<p><math>\lim_{x \rightarrow 2} \left( \frac{\sqrt{1 - \cos\{2(x-2)\}}}{x-2} \right)</math> (a) equals <math>-\sqrt{2}</math> (b) equals <math>\frac{1}{\sqrt{2}}</math> (c) does not exist (d) equals <math>\sqrt{2}</math>.</p> <p><b>Ans.</b> does not exist. <b>Reason.</b> <math>\lim_{x \rightarrow 2} \frac{\sqrt{1 - \cos(x-2)}}{x-2}</math></p> <p><math>\lim_{x \rightarrow 2} \frac{\sqrt{2}  \sin(x-2) }{x-2} = \sqrt{2}</math> or <math>-\sqrt{2}</math></p> <p>So limit does not exist.</p>				
46.	<p>If <math>A = \sin^2 x + \cos^4 x</math>, then for all real <math>x</math>: (a) <math>1 \leq A \leq 2</math> (b) <math>\frac{3}{4} \leq A \leq \frac{13}{16}</math> (c) <math>\frac{3}{4} \leq A \leq 1</math> (d) <math>\frac{13}{16} \leq A \leq 1</math>.</p> <p><b>Ans.</b> <math>\frac{3}{4} \leq A \leq 1</math> <b>Reason.</b> <math>A = \sin^2 x + \cos^4 x</math></p> <p><math>= \cos^4 x - \cos^2 x + 1</math></p> <p><math>= (\cos^2 x - 1/2)^2 + 3/4</math></p> <p><math>\therefore A \geq 3/4, A \leq (1 - 1/2)^2 + 3/4</math></p> <p><math>A \leq 1</math></p> <p><math>\therefore \frac{3}{4} \leq A \leq 1</math>.</p> <p>Option (c).</p>				
47.	<p>The lines <math>L_1 : y - x = 0</math> and <math>L_2 : 2x + y = 0</math> intersect the line <math>L_3 : y + 2 = 0</math> at P and Q respectively. The bisector of the acute angle between <math>L_1</math> and <math>L_2</math> intersects <math>L_3</math> at R. <b>Statement – 1:</b> The ratio PR : RQ equals <math>2\sqrt{2} : \sqrt{5}</math>. <b>Statement – 2:</b> In any triangle, bisector of an angle divides the triangle into two similar triangles. (a) Statement – 1 is true, Statement – 2 is false. (b) Statement – 1 is false, Statement – 2 is true (c) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1. (d) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not a correct explanation for Statement – 1.</p> <p><b>Ans.</b> Statement – 1 is true, Statement – 2 is false. <b>Reason.</b> Solving P(-2, -2), Q(1, -2)</p> <p>Bisector of acute angle between <math>y - x = 0</math> and <math>2x + y = 0</math></p> <p>is <math>(\sqrt{5} - 2\sqrt{2})x = (\sqrt{5} + \sqrt{2})y</math></p> <p>It intersect two line <math>y + 2 = 0</math> at R(<math>6+2\sqrt{10}</math>, -2)</p>				

	<p>Now <math>\frac{PR}{RQ} = \frac{8+2\sqrt{10}}{5+2\sqrt{10}} = \frac{2\sqrt{2}}{\sqrt{5}} \left( \frac{8+2\sqrt{10}}{5+2\sqrt{10}} \right) \frac{\sqrt{5}}{2\sqrt{2}}</math></p> $= \frac{2\sqrt{2}}{\sqrt{5}} \left( \frac{8\sqrt{5}+10\sqrt{2}}{8\sqrt{5}+10\sqrt{2}} \right) = \frac{2\sqrt{2}}{\sqrt{5}}$ <p>Statement – 1 is true, but Statement – 2 is false.</p>
48.	<p>The domain of the function <math>f(x) = \frac{1}{\sqrt{ x -x}}</math> is : (a) <math>(-\infty, 0)</math> (b) <math>(-\infty, \infty) - \{0\}</math> (c) <math>(-\infty, \infty)</math> (d) <math>(0, \infty)</math>.</p> <p><b>Ans.</b> <math>(-\infty, 0)</math> <b>Reason.</b> <math>f(x) = \frac{1}{\sqrt{ x -x}}</math></p> <p><math> x  - x &gt; 0</math>  <math> x  &gt; x</math>          Let <math>x &gt; 0</math>  <math>x &gt; x</math> (false)          Let <math>x &lt; 0</math>  <math>-x &gt; x</math>  <math>0 &gt; 2x</math>  <math>2x &lt; 0</math>  <math>x &lt; 0</math>  <math>-\infty &lt; x &lt; 0</math>  <math>x \in (-\infty, 0)</math></p>
49.	<p>If the angle between the line <math>x = \frac{y-1}{2} = \frac{z-3}{\lambda}</math> and the plane <math>x + 2y + 3z = 4</math> is <math>\cos^{-1} \left( \frac{\sqrt{5}}{\sqrt{14}} \right)</math>, then <math>\lambda</math> equals : (a) <math>2/5</math> (b) <math>5/3</math> (c) <math>2/3</math> (d) <math>3/2</math>.</p> <p><b>Ans.</b> <math>2/3</math> <b>Reason.</b> <math>(\lambda = 2/3)</math></p> <p><math>\frac{x}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}</math> <math>\cos^{-1} \frac{5}{\sqrt{14}}</math> is the angle between L and <math>\pi</math></p> <p><math>x + 2y + 3z = 4</math> (<math>\pi</math>) <math>\Rightarrow \sin^{-1} \left( \frac{3}{\sqrt{14}} \right)</math></p> <p><math>\theta = \sin^{-1} \left[ \frac{1+4+3\lambda}{\sqrt{14} + \sqrt{\lambda^2+5}} \right] = \sin^{-1} \left( \frac{3}{\sqrt{14}} \right)</math></p> <p><math>\Rightarrow \frac{5+3\lambda}{\sqrt{\lambda^2+5}} = 3</math></p> <p><math>\Rightarrow 5+3\lambda = 3\sqrt{\lambda^2+5}</math></p> <p>Squaring  <math>25 + 9\lambda^2 + 30\lambda = 9(\lambda^2 + 5)</math>  <math>\Rightarrow 30\lambda = 20</math>  <math>\Rightarrow \lambda = 2/3</math>          Option (c).</p>
50.	<p>The shortest distance between line <math>y - x = 1</math> and curve <math>x = y^2</math> is : (a) <math>\frac{8}{3\sqrt{2}}</math> (b) <math>\frac{4}{\sqrt{3}}</math> (c) <math>\frac{\sqrt{3}}{4}</math> (d) <math>\frac{3\sqrt{2}}{8}</math>.</p> <p><b>Ans.</b> <math>\frac{3\sqrt{2}}{8}</math> <b>Reason.</b> <math>y - x = 1 \Rightarrow y = x + 1 \dots (1)</math></p> <p style="text-align: center;"><math>(m = 1)</math></p> <p><math>x = y^2 \Rightarrow y^2 = 4(1/4)x</math> (<math>a = 1/4</math>)</p>



Shortest distance equation of tangent

$$y = mx + a/m$$

$$y = x + 1/4$$

... (2)

Distance between (1) to (2)

$$\frac{(1-1/4)}{\sqrt{1+1}} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

Option (d)

51. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after : (a) 20 months (b) 21 months (c) 18 months (d) 19 months.

**Ans.** 21 months **Reason.**  $(200 \times 3) + n/2 [480 + (n - 1) 40] = 11040$

$$\Rightarrow n^2 + 11n - 522 = 0$$

$$n = 18, -29$$

$$n + 3 = 18 + 3 = 21.$$

Option (b).

52. If the mean deviation about the median of the number  $a, 2a, \dots, 50a$  is 50, then  $|a|$  equals : (a) 4 (b) 5 (c) 2 (d) 3.

**Ans.** 4 **Reason.**  $a, 2a, 3a, \dots, 50a$

$$\text{Median} = \frac{25a + 26a}{2} = 25.5a$$

$$\text{M.D} = \frac{|a - 25.5a| + |2a - 25.5a| + \dots + |50a - 25.5a|}{50}$$

$$\Rightarrow \frac{2a[24.5 + 23.5 + \dots + (0.5)]}{50} = 50$$

$$\Rightarrow \frac{2a \cdot \frac{25}{2} [24.5 + 0.5]}{50} = 50 \Rightarrow a = 4$$

53. If  $\omega (\neq 1)$  is a cube root of unity, and  $(1 + \omega)^7 = A + B\omega$ . Then (A, B) equals: (a) (1, 0) (b) (-1, 1) (c) (0, 1) (d) (1, 1)

**Ans.** (1, 1) **Reason.**  $(1 + \omega)^7 = A + B\omega$

$$1 + 7\omega + 21\omega^2 + 35\omega^3 + 35\omega^4 + 21\omega^5 + 7\omega^6 + \omega^7$$

$$= 1 + 7\omega + 21\omega^2 + 35 + 35\omega + 21\omega^2 + 7 + \omega$$

$$= 43 + 43\omega + 42\omega^2$$

$$= 43(1 + \omega) + 42\omega^2$$

$$= 43(-\omega^2) + 42\omega^2$$

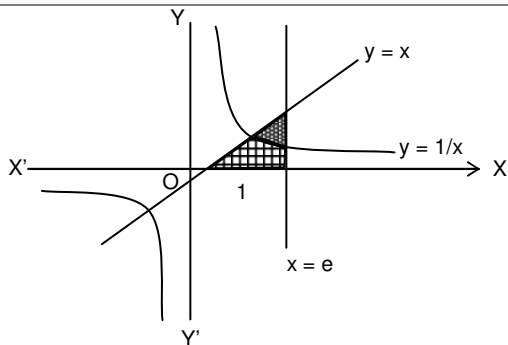
$$= -\omega^2$$

$$= -(-1 - \omega)$$

$$= 1 + \omega \Rightarrow A = 1, B = 1.$$

54. The area of the region enclosed by the curves  $y = x, x = e, y = 1/x$  and the positive x-axis is : (a) 3/2 square units (b) 5/2 square units (c) 1/2 square units (d) 1 square units

**Ans.** 3/2 square units **Reason.**



$$x = \frac{1}{x}$$

$$x^2 = 1$$

$$x = 1, -1$$

Area :

$$\int_0^1 x \, dx + \int_1^e \frac{1}{x} \, dx$$

$$= \left[ \frac{x^2}{2} \right]_0^1 + [\ln x]_1^e = \left( \frac{1}{2} - 0 \right) + (1 - 0) = \frac{3}{2} \text{ sq. unit.}$$

55. The number of values of k for which the linear equations  $4x + ky + 2z = 0$ ,  $kx + 4y + z = 0$ ,  $2x + 2y + z = 0$ , possess a non-zero solution is : (a) 1 (b) zero (c) 3 (d) 2.

**Ans. 2 Reason.** For non-zero solution

$$\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow k^2 - 6k + 8 = 0$$

$$\Rightarrow k = \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm 2}{2} = 4, 2$$

Hence two values of k.

- 56.

The values of p and q for which the function  $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$  is continuous for all x

in R, are : (a)  $P = -\frac{3}{2}$ ,  $q = \frac{1}{2}$  (b)  $P = \frac{1}{2}$ ,  $q = \frac{3}{2}$ , (c)  $P = \frac{1}{2}$ ,  $q = -\frac{3}{2}$  (d)  $P = \frac{5}{2}$ ,  $q = \frac{1}{2}$ .

**Ans.**  $P = -\frac{3}{2}$ ,  $q = \frac{1}{2}$  **Reason.** LHL =  $\lim_{x \rightarrow 0^-} \frac{\sin(P+1)x + \sin x}{x}$

$$= \lim_{h \rightarrow 0} \frac{\sin(P+1)h + \sin h}{h} = P + 1 + 1 = P + 2.$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} = \lim_{h \rightarrow 0} \frac{\sqrt{h+h^2} - \sqrt{h}}{h\sqrt{h}}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h}$$

$$= \frac{1}{2}$$

	<p>Now <math>P + 2 = q</math> and <math>q = \frac{1}{2}</math>                  Since <math>f(0) = q</math>                  Hence <math>P = -\frac{3}{2}</math>.</p>
57.	<p><b>Statement – 1</b> : The number of ways of distributing 10 identical balls in 4 distinct boxes which that no box is empty is <math>{}^9C_3</math>. <b>Statement – 2</b> : The number of ways of choosing any 3 places from 9 different places is <math>{}^9C_3</math>. (a) Statement – 1 is true, Statement – 2 is false. (b) Statement – 1 is false, Statement – 2 is true (c) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1. (d) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not a correct explanation for Statement – 1.</p> <p><b>Ans.</b> Statement – 1 is false, Statement – 2 is true. <b>Reason.</b> The number of ways of distributing 10 identical balls in 4 distinct boxes  <math>= {}^{n+r-1}C_{r-1} = {}^{10+4-1}C_{4-1} = {}^{13}C_3</math>                  Hence statement -1 is false.                  The no. of ways of selecting 3 places from 9 places = <math>{}^9C_3</math>                  Hence statement – 2 is true.</p>
58.	<p>Equation of the ellipse whose axes are the axes of coordinates and which passes through the point <math>(-3, 1)</math> and has eccentricity <math>\sqrt{\frac{2}{5}}</math> is : (a) <math>3x^2 + 5y^2 - 15 = 0</math> (b) <math>5x^2 + 3y^2 - 32 = 0</math> (c) <math>3x^2 + 5y^2 - 32 = 0</math> (d) <math>5x^2 + 3y^2 - 48 = 0</math>.</p> <p><b>Ans.</b> <math>3x^2 + 5y^2 - 32 = 0</math> <b>Reason.</b> Let <math>\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1</math>  <math>\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1</math>  <math>\Rightarrow \frac{x^2}{a^2} + \frac{5y^2}{3a^2} = 1 \Rightarrow c^2 = \frac{2a^2}{5}</math>  <math>(-3,1) \frac{9}{a^2} + \frac{5}{3a^2} = 1</math>  <math>\Rightarrow a^2 = \frac{32}{3} \text{ \&amp; } b^2 = \frac{3a^2}{5} = \frac{32}{5}</math>  <math>\therefore</math> equation is <math>\frac{3x^2}{32} + \frac{5y^2}{32} = 1</math>  <math>\Rightarrow 3x^2 + 5y^2 - 32 = 0</math></p>
59.	<p>Let <math>I</math> be the purchase value of an equipment and <math>V(t)</math> be the value after it has been used for <math>t</math> years. The value <math>V(t)</math> depreciates at a rate given by differential equation <math>\frac{dV(t)}{dt} = -k(T - t)</math>, where <math>k &gt; 0</math> is a constant and <math>T</math> is the total life in years of the equipment. Then the scrap value <math>V(T)</math> of the equipment is : (a) <math>I - \frac{k(T-t)^2}{2}</math> (b) <math>e^{-kT}</math> (c) <math>T^2 - \frac{I}{k}</math> (d) <math>I - \frac{kT^2}{2}</math>.</p> <p><b>Ans.</b> <math>I - \frac{kT^2}{2}</math> <b>Reason.</b> <math>\frac{dV(t)}{dt} = -K(T - t) \quad K &gt; 0</math>  <math>\int dV(t) = -K \int (T - t) dt</math>  <math>V(t) = \frac{-K(T-t)^2}{-2} + C</math>  <math>= \frac{K(T-t)^2}{2} + C</math>                  for <math>t = T</math>  <math>V(T) = 0 + C</math>  <math>V(T) = C</math>                  For <math>t = 0</math></p>

$$V(0) = \frac{KT^2}{2} + C$$

We know that

$$V(0) = 1$$

$$1 = \frac{KT^2}{2} + C$$

$$C = 1 - \frac{KT^2}{2}$$

$$\text{Now } V(T) = 1 - \frac{KT^2}{2}$$

Option : (d)

60.

If  $\frac{dy}{dx} = y + 3 > 0$  and  $y(0) = 2$ , then  $y(\ln 2)$  is equal to : (a) 13 (b) -2 (c) 7 (d) 5.

**Ans. 7 Reason.**  $\frac{dy}{dx} = (y + 3) > 0$   $y(0) = 2$

$$\int \frac{dy}{y+3} = \int dx \quad y(\ln 2) = \underline{\quad}$$

$$\ln(y+3) = x + \ln C$$

$$\ln\left(\frac{y+3}{C}\right) = x$$

$$y+3 = C e^x$$

$$\text{for } (x=0), y=2 \Rightarrow 2+3 = C(e^0) = C$$

$$C = 5$$

$$y+3 = 5e^x$$

$$\text{for } x = \ln 2$$

$$y+3 = 5(e^{\ln 2}) = (5 \times 2)$$

$$= 10$$

$$y = 10 - 3 = 7.$$

### PHYSICS

61.

A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in mirror of the first one is : (a) 10 m/s (b) 15 m/s (c)  $\frac{1}{10}$  m/s (d)  $\frac{1}{15}$  m/s

**Ans. 15 m/s Reason:**  $V_1 = \left(\frac{F}{u+F}\right)^2 V_0$

$$= \left(\frac{20}{280+20}\right)^2 \times 15 = \left(\frac{1}{15}\right)^2 \times 15$$

$$= \frac{1}{15} \text{ m/s}$$

62.

The half life of a radioactive substance is 20 minutes. The approximate time interval ( $t_2 - t_1$ ) between the time  $t_2$  when  $\frac{2}{3}$  of it has decayed and time  $t_1$  when  $\frac{1}{3}$  of it had decayed is : (a) 20 min (b) 28 min (c) 7 min (d) 14 min.

**Ans. 20 min Reason.**  $\frac{2}{3} = \left(\frac{1}{2}\right)^{\frac{t_2}{20}}$

	$\frac{1}{3} = \left(\frac{1}{2}\right)^{\frac{t_1}{20}}$ $\Rightarrow 2 = \left(\frac{1}{2}\right)^{\frac{t_2 - t_1}{20}}$ $\Rightarrow \frac{t_2}{20} - \frac{t_1}{20} = 1$ $\Rightarrow t_2 - t_1 = 20.$
63.	<p>A boat is moving due east in a region where the earth's magnetic field is <math>5.0 \times 10^{-5} \text{ NA}^{-1} \text{ m}^{-1}</math> due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is <math>1.50 \text{ ms}^{-1}</math>, the magnitude of the induced emf in the wire of aerial is : (a) 0.50 mV (b) 0.15 mV (c) 1 mV (d) 0.75 mV.</p> <p><b>Ans.</b> 0.15 mV <b>Reason.</b> <math>e = Blv</math>  <math>= 5 \times 10^{-5} \times 2 \times 1.5</math>  <math>= 15 \times 10^{-5} \text{ V}</math>  <math>= 0.15 \text{ mV}.</math></p>
64.	<p>The transverse displacement <math>y(x, t)</math> of a wave on a string is given by <math>y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)}</math>. This represents a : (a) standing wave of frequency <math>\sqrt{b}</math> (b) standing wave of frequency <math>\frac{1}{\sqrt{b}}</math> (c) wave moving in +x direction with speed <math>\sqrt{\frac{a}{b}}</math> (d) wave moving in -x direction with <math>\sqrt{\frac{b}{a}}</math>.</p> <p><b>Ans.</b> wave moving in -x direction with <math>\sqrt{\frac{b}{a}}</math>. <b>Reason.</b> <math>y = e^{-(\sqrt{ax})^2 + (\sqrt{bt})^2 + 2(\sqrt{ax})(\sqrt{bt})}</math>  <math>= e^{-[\sqrt{ax} + \sqrt{bt}]^2}</math>  wave speed = <math>\frac{\text{coeff. of } t}{\text{coeff. of } x} = \frac{\sqrt{b}}{\sqrt{a}}.</math></p>
65.	<p>A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is <math>v</math>, the total area around the fountain that gets wet is : (a) <math>\frac{\pi v^4}{2g^2}</math> (b) <math>\pi \frac{v^2}{g^2}</math> (c) <math>\pi \frac{v^2}{g}</math> (d)</p> <p><math>\pi \frac{v^4}{g^2}</math></p> <p><b>Ans.</b> <math>\pi \frac{v^4}{g^2}</math> <b>Reason.</b> <math>R_{\max} = \frac{v^2}{g}</math>  <math>A = \pi R^2 = \pi \frac{v^4}{g^2}</math></p>
66.	<p>Two particles are executing simple harmonic motion of the same amplitude <math>A</math> and frequency <math>\omega</math> along the x-axis. Their mean position is separated by distance <math>X_0 (X_0 &gt; A)</math>. If the maximum separation between them is <math>(X_0 + A)</math>, the phase difference between their motion is : (a) <math>\frac{\pi}{4}</math> (b) <math>\frac{\pi}{6}</math> (c) <math>\frac{\pi}{2}</math> (d) <math>\frac{\pi}{3}</math></p> <p><b>Ans.</b> <math>\pi/2</math> <b>Reason.</b> If one is at mean position, then other is at extreme position.</p>
67.	<p>This question has Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best described the two statements.</p> <p><b>Statement – 1</b>  A metallic surface is irradiated by a monochromatic light of frequency <math>\nu &gt; \nu_0</math> (the threshold frequency). The maximum kinetic energy and the stopping potential are <math>K_{\max}</math> and <math>V_0</math> respectively. If</p>

	<p>the frequency incident on the surface is doubled, both <math>K_{\max}</math> and <math>V_0</math> are also doubled.</p> <p><b>Statement – 2</b></p> <p>The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.</p> <p>(a) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not the correct explanation of statement – 1. (b) Statement – 1 is false, Statement – 2 is true. (c) Statement – 1 is true, Statement – 2 is false. (b) Statement – 1 is true, Statement – 2 is true, Statement – 2 is the correct explanation of Statement – 1.</p> <p><b>Ans.</b> Statement – 1 is false, Statement – 2 is true.</p>
68.	<p>Let the <math>x - z</math> plane be the boundary between two transparent media. Medium 1 in <math>z \geq 0</math> has a refractive index of <math>\sqrt{2}</math> and medium 2 with <math>z &lt; 0</math> has a refractive index of <math>\sqrt{3}</math>. A ray of light in medium 1 given by the vector <math>\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}</math> is incident on the plane of separation. The angle of refraction in medium 2 is : (a) <math>60^\circ</math> (b) <math>75^\circ</math> (c) <math>30^\circ</math> (d) <math>45^\circ</math>.</p> <p><b>Ans.</b> <math>30^\circ</math> <b>Reason.</b> <math>\sqrt{2} \frac{\sqrt{(6\sqrt{3})^2 + (10)^2}}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (10)^2}} = \sqrt{3} \sin \beta</math></p> $\frac{\sqrt{2}}{\sqrt{3}} \frac{\sqrt{208}}{\sqrt{20}} = \sin \beta \Rightarrow \beta = 30^\circ.$
69.	<p>A Carnot engine operating between temperatures <math>T_1</math> and <math>T_2</math> has efficiency <math>\frac{1}{6}</math>. When <math>T_2</math> is lowered by 62 K, its efficiency increases to <math>\frac{1}{3}</math>. Then <math>T_1</math> and <math>T_2</math> are, respectively : (a) 330 K and 268 K (b) 310 K and 248 K (c) 372 K and 310 K (d) 372 K and 330 K.</p> <p><b>Ans.</b> 372 K and 310 K <b>Reason.</b> <math>1 - \frac{T_2}{T_1} = \frac{1}{6} \dots (1)</math></p> $1 - \frac{(T_2 - 62)}{T_1} = \frac{1}{3} \dots (2)$ <p>Solving <math>T_2 = 372\text{K}</math> and <math>T_1 = 310\text{K}</math>.</p>
70.	<p>Energy required for the electron excitation in <math>\text{Li}^{++}</math> from the first to the third Bohr orbit is : (a) 108.8 eV (b) 122.4 eV (c) 12.1 eV (d) 36.3 eV</p> <p><b>Ans.</b> 108.8 eV <b>Reason.</b> <math>\Delta E = E_0 z^2 (1 - 1/9)</math></p> $= 13.6 \times 9 \times 8/9 = 108.8 \text{ eV}.$
71.	<p>A resistor 'R' and <math>2\mu\text{F}</math> capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. (<math>\log_{10} 2.5 = 0.4</math>) (a) <math>2.7 \times 10^6 \Omega</math> (b) <math>3.3 \times 10^7 \Omega</math> (c) <math>1.3 \times 10^4 \Omega</math> (d) <math>1.7 \times 10^5 \Omega</math>.</p> <p><b>Ans.</b> <math>2.7 \times 10^6 \Omega</math> <b>Reason.</b> <math>80 = 200 e^{-\frac{5}{2R} \times 10^6}</math></p> $\frac{5}{2R} 10^6 = 0.4 \times 2.3$ $R = \frac{50}{8 \times 2.3} = 2.7 \times 10^6.$
72.	<p>A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats <math>\gamma</math>. It is moving with speed <math>v</math> and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by : (a) <math>\frac{\gamma M v^2}{2R}</math> K (b) <math>\frac{(\gamma - 1)}{2R} M v^2</math> K (c) <math>\frac{(\gamma - 1)}{2(\gamma + 1)R} M v^2</math> K (d) <math>\frac{(\gamma - 1)}{2\gamma R} M v^2</math> K</p>

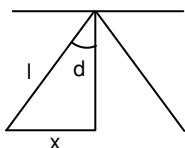
	<p><b>Ans.</b> <math>\frac{(\gamma-1)}{2R} Mv^2 K</math> <b>Reason.</b> <math>\frac{1}{2} mv^2 = c_v dT</math></p> $\frac{1}{2} mv^2 = \frac{R}{\gamma-1} dT$ $dT = \frac{1}{2} mv^2 \left( \frac{\gamma-1}{R} \right) = \frac{(\gamma-1)}{2R} Mv^2 K$
73.	<p>Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = <math>0.03 \text{ Nm}^{-1}</math>) : (a) <math>2\pi \text{ mJ}</math> (b) <math>0.4\pi \text{ mJ}</math> (c) <math>4\pi \text{ mJ}</math> (d) <math>0.2\pi \text{ mJ}</math></p> <p><b>Ans.</b> <math>0.2\pi \text{ mJ}</math> <b>Reason.</b> <math>W = TdA</math>  <math>= 0.03 \times 4\pi \times 16 \times 10^{-4} \text{ J}</math>  <math>= 1.92\pi \times 10^{-4} \text{ J}</math>  <math>\approx 2\pi \times 10^{-4} \text{ J}</math>  <math>\approx 0.2\pi \times 10^{-3} \text{ J}.</math></p>
74.	<p>A fully charged capacitor C with initial charge <math>q_0</math> is connected to a coil of self inductance L, at <math>t = 0</math>. The time at which the energy is stored equally between the electric and the magnetic field is : (a) <math>2\pi\sqrt{LC}</math> (b) <math>\sqrt{LC}</math> (c) <math>\pi\sqrt{LC}</math> (d) <math>\frac{\pi}{4}\sqrt{LC}</math></p> <p><b>Ans.</b> <math>\pi\sqrt{LC}</math> <b>Reason.</b> The capacitor is fully discharged in time <math>2\pi\sqrt{LC}</math>.  So the time at which the energy is stored equally between the electric and the magnetic field is <math>\pi\sqrt{LC}</math>.</p>
75.	<p><b>Direction :</b>  The question has paragraph followed by two statements, <b>Statement – 1</b> and <b>Statement – 2</b>. Of the given four alternatives after the statements, choose the one that describes the statements.</p> <p>A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.</p> <p><b>Statement – 1 :</b>  When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of <math>\pi</math>.</p> <p><b>Statement – 2 :</b>  The centre of the interference pattern is dark.</p> <p>(a) Statement -1 is true, Statement -2 is true and Statement -2 is not the correct explanation of Statement -1. (b) Statement -1 is false, Statement -2 is true. (c) Statement -1 is true, Statement -2 is false. (d) Statement -1 is true, Statement -2 is true and Statement -2 is the correct explanation of statement -1.</p> <p><b>Ans.</b> Statement -1 is true, Statement -2 is true and Statement -2 is the correct explanation of statement -1. <b>Reason.</b> Facts.</p>
76.	<p>A screw gauge gives the following reading when used to measure the diameter of a wire. Main scale reading : 0 mm. Circular scale reading : 52 divisions. Given that 1 mm on main scale corresponds to 100 divisions of the circular scale. The diameter of wire from the above data is : (a) 0.026 cm (b) 0.005 cm (c) 0.52 cm (d) 0.052 cm.</p> <p><b>Ans.</b> 0.052 cm <b>Reason.</b> 100 div. is equal to <math>\frac{1}{10}</math> cm.</p> <p>1 div. is equal to <math>\frac{1}{1000}</math> cm.</p> <p>52 div. is equal to <math>\frac{52}{1000} = 0.052 \text{ cm}.</math></p>
77.	<p>Three perfect gases at absolute temperatures <math>T_1</math>, <math>T_2</math> and <math>T_3</math> are mixed. The masses of molecules are <math>m_1</math>, <math>m_2</math> and <math>m_3</math> and the number of molecules are <math>n_1</math>, <math>n_2</math> and <math>n_3</math> respectively. Assuming no loss of energy, the final temperature of the mixture is : (a) <math>\frac{n_1 T_1^2 + n_2 T_2^2 + n_3 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}</math> (b) <math>\frac{n_1^2 T_1^2 + n_2^2 T_2^2 + n_3^2 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}</math> (c)</p>

	$\frac{(T_1 + T_2 + T_3)}{3} \quad (d) \quad \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$ <p><b>Ans.</b> <math>\frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}</math> <b>Reason.</b> <math>(n_1 + n_2 + n_3) CvT = n_1 CvT_1 + n_2 CvT_2 + n_3 CvT_3</math></p> $T = \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$
78.	<p>The electrostatic potential inside a charged spherical ball is given by <math>\phi = ar^2 + b</math> where <math>r</math> is the distance from the centre; <math>a, b</math> are constants. Then the charge density inside the ball is : (a) <math>-24 \pi a \epsilon_0</math> (b) <math>-6 a \epsilon_0</math> (c) <math>-24 \pi a \epsilon_0 r</math> (d) <math>-6 a \epsilon_0 r</math>.</p> <p><b>Ans.</b> <math>-6 a \epsilon_0</math> <b>Reason.</b> <math>v = ar^2 + b</math></p> $E = -\frac{dv}{dr} = -2ar$ $E (4\pi r^2) = \theta/\epsilon_0$ $(-2ar) (4 \pi r^2) = \theta/\epsilon_0$ $\frac{Q}{v} = \frac{-8\pi\epsilon_0 ar^3}{\frac{4}{3}\pi r^3} = -6 \epsilon_0 a$
79.	<p>This question has Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements.</p> <p><b>Statement – 1 :</b></p> <p>Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.</p> <p><b>Statement – 2 :</b></p> <p>The state of ionosphere varies from hour to hour/day to day and season to season.</p> <p>(a) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not the correct explanation of Statement – 1. (b) Statement – 1 is false, Statement – 2 is true, (c) Statement – 1 is true, Statement – 2 is false (d) Statement – 1 is true, Statement – 2 is true, Statement – 2 is the correct explanation of Statement – 1.</p> <p><b>Ans.</b> Statement – 1 is true, Statement – 2 is false <b>Reason.</b> Facts</p>
80.	<p>A mass <math>m</math> hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass <math>m</math> and radius <math>R</math>. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass <math>m</math>, if the string does not slip on the pulley, is : (a) <math>\frac{2}{3}g</math> (b) <math>\frac{g}{3}</math> (c) <math>\frac{3}{2}g</math> (d) <math>g</math>.</p> <p><math>\frac{2}{3}g</math> <b>Reason.</b> <math>mg - T = ma \dots\dots(1)</math></p> $Z = I \propto$ $T \times R = \frac{1}{2} m R^2 \left( \frac{a}{R} \right)$ $\Rightarrow T = \frac{ma}{2}$ <p>putting in (1)</p> $mg - \frac{ma}{2} = ma$ $3 \frac{ma}{2} = mg \Rightarrow a = \frac{2g}{3}$
81.	<p>A current <math>I</math> flows in an infinitely long wire with cross section in the form of a semi-circular ring of radius</p>

	<p>R. The magnitude of the magnetic induction along its axis is : (a) <math>\frac{\mu_0 I}{2\pi R}</math> (b) <math>\frac{\mu_0 I}{4\pi R}</math> (c) <math>\frac{\mu_0 I}{\pi^2 R}</math> (d) <math>\frac{\mu_0 I}{2\pi^2 R}</math></p> <p><b>Ans.</b> <math>\frac{\mu_0 I}{\pi^2 R}</math> <b>Reason.</b> <math>\frac{\mu_0 I'}{2\pi R} = \frac{\mu_0}{2\pi R} \left( \frac{IRd\theta}{2\pi R} \right)</math></p> $B = \frac{\mu_0}{2\pi^2 R^2} IR \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = \frac{\mu_0 IR(2)}{2\pi^2 R^2}$ $= \frac{\mu_0 I}{\pi^2 R}$
82.	<p>If a wire is stretched to make it 0.1% longer, its resistance will : (a) decrease by 0.2% (b) decrease by 0.05% (c) increase by 0.05% (d) increase by 0.2%.</p> <p><b>Ans.</b> increase by 0.2%. <b>Reason.</b> <math>R = \frac{\rho l^2}{v}</math></p> $\frac{dR}{R} \% = 2 \left( \frac{dl}{l} \% \right) = 2(0.1\%)$ $= 0.2 \%$
83.	<p>A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc : (a) continuously increases (b) first increases and then decreases (c) remains unchanged (d) continuously decreases.</p> <p><b>Ans.</b> first increases and then decreases <b>Reason.</b> because moment of inertia 1st decrease and then increase.</p>
84.	<p>100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K) : (a) 84 kJ (b) 2.1 kJ (c) 4.2 kJ (d) 8.4 kJ.</p> <p><b>Ans.</b> 8.4 kJ. <b>Reason :</b> <math>d\theta = msdT = 100 \times 10^{-3} \times 8148 \times 20 = 8148 \times 2 = 8368 \text{ J} = 8.368 \text{ J} = 8.4 \text{ kJ}</math>.</p>
85.	<p>An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by : <math>\frac{dv}{dt} = -2.5\sqrt{v}</math>, where <math>v</math> is the instantaneous speed. The time taken by the object, to come to rest, would be : (a) 4s (b) 8s (c) 1s (d) 2s.</p> <p><b>Ans.</b> 2s. <b>Reason :</b> <math>t = \int_v^0 -\frac{dW}{2.5\sqrt{v}} = -\frac{1}{2.5} [2\sqrt{v}]_v^0</math></p> $= \frac{2\sqrt{v}}{2.5} = \frac{2\sqrt{6.25}}{2.5} = 2$
86.	<p>Water is flowing continuously from a tap having an internal diameter <math>8 \times 10^{-3}</math> m. The water velocity as it leaves the tap is <math>0.4 \text{ ms}^{-1}</math>. The diameter of the water stream at a distance <math>2 \times 10^{-1}</math> m below the tap is close to : (a) <math>9.6 \times 10^{-3}</math> m (b) <math>3.6 \times 10^{-3}</math> m (c) <math>5.0 \times 10^{-3}</math> m (d) <math>7.5 \times 10^{-3}</math> m.</p> <p><b>Ans.</b> <math>3.6 \times 10^{-3}</math> m <b>Reason :</b> <math>A_1 v_1 = A_2 v_2</math></p> $d_1^2 v_1 = d_2^2 v_2$ $\left( \frac{d_2}{d_1} \right)^2 = \frac{v_1}{v_2} = \frac{v_1}{\sqrt{v_1^2 + 2gh}} = \frac{0.4}{\sqrt{0.4^2 + 2 \times 10 \times 0.2}} = 0.2$ $d_2/d_1 = 0.44, d_2 = 8 \times 10^{-3} \times 0.44 = 3.6 \times 10^{-3} \text{ m}$
87.	<p>Two identical charged spheres suspended from a common point by two massless strings of length <math>l</math> are initially a distance <math>d</math> (<math>d \ll l</math>) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity <math>v</math>. Then as a function of distance <math>x</math> between them, (a) <math>v \propto x^{1/2}</math> (b) <math>v \propto x</math> (c) <math>v \propto x^{-1/2}</math> (d) <math>v \propto x^{-1}</math>.</p> <p><b>Ans.</b> <math>v \propto x^{-1/2}</math>. <b>Reason :</b> <math>T \sin \theta = kq^2 / x^2</math>, <math>T \cos \theta = mg</math>.</p> $\tan \theta = \frac{mgx^2}{kq^2}, \frac{q^2}{4\pi\epsilon_0 x^2} = mg \frac{x}{2L}$ $q^2 = \frac{2\pi\epsilon_0 mg}{L} x^3$

$$2q \frac{dq}{dt} = \frac{2\pi\epsilon_0 mg}{1} 3x^2 \frac{dx}{dt}$$

$$\Rightarrow \frac{dq}{dt} = \frac{3}{2} \sqrt{\frac{2\pi\epsilon_0 mg}{1}} x^{1/2} \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = x^{-1/2}$$



88. A mass  $M$ , attached to a horizontal spring, executes S.H.M. with amplitude  $A_1$ . When the mass  $M$  passes through its mean position then a smaller mass  $m$  is placed over it and both of them move together with amplitude  $A_2$ . The ratio of  $\left(\frac{A_1}{A_2}\right)$  is : (a)  $\left(\frac{M}{M+m}\right)^{1/2}$  (b)  $\left(\frac{M+m}{M}\right)^{1/2}$  (c)  $\frac{M}{M+m}$  (d)

$$\frac{M+m}{M}$$

**Ans.**  $\left(\frac{M+m}{M}\right)^{1/2}$  . **Reason :**  $m_1 v_1 = m_2 v_2$

$$m_1 \omega_1 A_1 = m_2 \omega_2 A_2$$

$$m_1 \sqrt{\frac{k}{m_1}} A_1 = m_2 \sqrt{\frac{k}{m_2}} A_2$$

$$\sqrt{m_1} A_1 = \sqrt{m_2} A_2 \quad \frac{A_1}{A_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{M+m}{M}}$$

89. Two bodies of masses  $m$  and  $4m$  are placed at a distance  $r$ . The gravitational potential at a point on the line joining them where the gravitational field is zero is (a)  $-\frac{6Gm}{r}$  (b)  $-\frac{9Gm}{r}$  (c) zero (d)

$$-\frac{4Gm}{r}$$

**Ans.**  $-\frac{9Gm}{r}$  . **Reason :**  $\frac{G \times 4m}{x^2} = \frac{Gm}{(r-x)^2} \Rightarrow x = 2r/3$

$$V = \frac{4Gm}{2r/3} + \frac{Gm}{r/3} = \frac{6Gm}{r} + \frac{3Gm}{r} = \frac{9Gm}{r} = -\frac{9Gm}{r}$$

90. A pulley of radius  $2m$  is rotated about its axis by a force  $F = (20t - 5t^2)$  newton (where  $t$  is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is  $10 \text{ kg m}^2$ , the number of rotations made by the pulley before its direction of motion is reversed, is : (a) more than 6 but less than 9. (b) more than 9 (c) less than 3 (d) more than 3 but less than 6.

**Ans.** more than 3 but less than 6 **Reason :**  $Fr = I\alpha \Rightarrow (20t - 5t^2)^2 = 10\alpha$

$$\Rightarrow d\omega = (4t - t^2) dt \Rightarrow \omega = 2t^2 - t^3/3$$

$$\text{Putting } \omega = 0 \Rightarrow t = 6 \text{ sec}$$

$$d\theta/dt = \omega = 2t^2 - t^3/3$$

$$d\theta = (2t^2 - t^3/3) dt$$

$$x = \theta/2\pi = 36/2\pi = 5.72$$

$$\theta = 2t^2/3 - t^4/12$$

$$\text{Putting } t = 6 \Rightarrow \theta = 36.$$

$$\text{Number of revolution} = \frac{36}{2\pi} = \frac{18}{\pi}$$