IIT JEE SOLUTION PAPER-2

MATHEMATICS

	SINGLE CORRECT ANSWER TYPE	ANS
41.	If $\lim_{x\to 0} [1+x\ \ln(1+b)^2]^{\frac{1}{x}} = 2\,b\,\sin^2\theta$, b > θ and θ ∈ (-π, π] then the value of θ is (a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$.	D
42.	Let f : [-1, 2] → [0, ∞) be a continuous function such that f(x) = f(1 – x) for all x∈[-1, 2]. Let $R_1 = \int_{-1}^{2} x\,f(x)\,dx$, and R_2 be the area of the region bounded by y = f(x), x =-1, x = 2, and the x-axis. Then (a) $R_1 = 2R_2$ (b) $R_1 = 3R_2$ (c) $2R_1 = R_2$ (d) $3R_1 = R_2$.	C
43.	Let $f(x) = x^2$ and g(x) = sin x for all x∈ ℝ. Then the set of all x satisfying (f o g o g o f) (x) = (g o g o f) (x), where (f o g) (x) = f(g(x)), is (a) $\pm\sqrt{n\pi}$, n∈ {0, 1, 2,….} (b) $\pm\sqrt{n\pi}$, n∈ {1, 2,….} (c) $\frac{\pi}{2}$ +2nπ, n∈ {…., -2, -1, 1, 1, 2, …..} (d) 2nπ, n∈ {…., -2, -1, 0, 1, 2, ….}	A
44.	Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1.3. Then the locus of P is (a) $x^2 = y$ (b) $y^2 = 2x$ (c) $y^2 = x$ (d) $x^2 = 2y$.	C
45.	Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is (a) $\sqrt{\frac{5}{2}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$	B
46.	A value of b for which the equations $\begin{matrix} x^2+bx-1=0 \\ x^2+x+b=0 \end{matrix}$ have one root in common is (a) $-\sqrt{2}$ (b) $-i\sqrt{3}$ (c) $i\sqrt{5}$ (d) $\sqrt{2}$.	B
47.	Let ω ≠ 1 be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set S is (a) 2 (b) 6 (c) 4 (d) 8.	A
48.	The circle passing through the point (-1, 0) and touching the y-axis at (0, 2) also passes through the point (a) $\left(-\frac{3}{2},0\right)$ (b) $\left(-\frac{5}{2},0\right)$ (c) $\left(-\frac{3}{2},\frac{5}{2}\right)$ (d) (-4, 0)	D

MULTI CORRECT ANSWER TYPE	

49.			If $f(x) = \begin{cases} -x-\frac{\pi}{2}, & x \le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x-1, & 0 < x \le 1 \\ \ln x & x > 1 \end{cases}$, then (a) f(x) is continuous at $x = -\frac{\pi}{2}$ (b) f(x) is not differentiable at x = 0 (c) f(x) is differentiable at x =1 (d) f(x) is differentiable $x = -\frac{3}{2}$	**A/B/C/D**
50.			Let $f : (0, 1) \to \mathbb{R}$ be defined by $f(x) = \frac{b-x}{1-bx}$, where b is a constant such that 0 < b < 1. Then (a) f is not invertible on (0, 1) (b) $f \ne f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$ (c) $f = f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$ (d) f^{-1} is differentiable on (0, 1)	**C/D**
51.			Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by (A) y – x + 3 = 0 (B) y + 3x – 33 = 0 (C) y + x – 15 = 0 (D) y – 2x + 12 = 0.	**A/B/D**
52.			Let E and F be two independent events. The probability that exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If P(T) denotes the probability of occurrence of the event T, the (A) P(E) = 4/5, P(F) = 3/5 (B) P(E) = 1/5, P(F) = 2/5 (C) P(E) = 2/5, P(F) = 1/5 (D) P(E) = 3/5, P(F) = 4/5.	**A/D**
53.			Let $\vec{a} = -\hat{i}-\hat{k}, \vec{b} = -\hat{i}+\hat{j}$ and $\vec{c} = \hat{i}+2\hat{j}+3\hat{k}$ be the three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}\,.\,\vec{a} = 0$, then the value of $\vec{r}\,.\,\vec{b}$ is	**9**
54.			The straight line 2x – 3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts. If $S = \left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$, then the number of point (s) in S lying inside the smaller part is	**2**
55.			Let $\omega = e^{i\pi/3}$ and a, b, c, x, y, z be non – zero complex numbers such that : a + b + c = x, $a + b\omega + c\omega^2 = y$, $a + b\omega^2 + c\omega = z$. Then the value of $\frac{\lvert x\rvert^2+\lvert y\rvert^2+\lvert z\rvert^2}{\lvert a\rvert^2+\lvert b\rvert^2+\lvert c\rvert^2}$ is	**3**
56.			The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is	**2**
57.			Let y' (x) + y(x)g'(x) = g(x)g'(x), y(0) = 0, $x \in \mathbb{R}$, where f'(x) denotes $\frac{df(x)}{dx}$ and g(x) is a given non – constant differentiable function on $\mathbb{R}$ with g(0) = g(2) = 0. Then the value of y(2) is	**0**

58. Let M be a 3 × 3 matrix satisfying $M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$, $M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$, and $M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$. Then the sum of the diagonal entries of M is

Answer: 9

59. Match the statements given in Column – I with the values given in Column – II.

Column – I	Column – II
(A) If $\vec{a} = \hat{j}+\sqrt{3}\hat{k}$, $\vec{b} = -\hat{j}+\sqrt{3}\hat{k}$, and $\vec{c} = 2\sqrt{3}\hat{k}$ a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p) $\pi / 6$
(B) If $\int_a^b (f(x)-3x)dx = a^2-b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q) $2\pi / 3$
(C) The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)\, dx$ is	(r) $\pi / 3$
(D) The maximum value of $\left\lvert \mathrm{Arg}\left(\frac{1}{1-z}\right)\right\rvert$ for $\lvert z\rvert = 1$, $z \neq 1$ is given by	(s) π
	(t) $\pi / 2$

Answer: AQ BP CS DT

60. Match the statements given in Column – I with the intervals / union of intervals given in Column – II.

Column – I	Column – II
(A) The set $\left\{\mathrm{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, \lvert z\rvert = 1, z \neq \pm 1\right.$ is	(p) $(-\infty, 1) \cup (1, \infty)$
(B) The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q) $(-\infty, 0) \cup (0, \infty)$
(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\right\}$ is	(r) $[2, \infty)$
(D) If $f(x) = x^{3/2}$ (3x 10), $x \ge 0$, then f(x) is increasing in	(s) $(-\infty, -1] \cup [1, \infty)$
	(t) $(-\infty, 0] \cup [2, \infty)$

Answer: AS BT CR DR