IIT JEE Qn. _2011 Dt. 11.04.11

SOLUTIONS (Paper – 2)

1. **Ans.** (A)
2. **Ans.** (C)
3. **Ans.** (B)
4. **Ans.** (B)
5. **Ans.** (D)
6. **Ans.** (C)
7. **Ans.** (A)
8. **Ans.** (D).
9. **Ans.** (A, D)
10. **Ans.** (ABCD)
11. **Ans.** (ABC)
12. **Ans.** (BCD)
13. **Ans.** 8
14. **Ans.** 6
15. **Ans.** 4
16. **Ans.** 6
17. **Ans.** 7
18. **Ans.** 8
19. **Ans.** A $\to$p, r, s B $\to$ r, s C $\to$ t, D $\to$ q, r
20. **Ans.** A $\to$ r, s, t B $\to$ s, p C $\to$ s, r, D $\to$ q, r
21. **Ans.** (C)
22. **Ans.** (D) **Reason.** $mV = mV_1 + mV_2$

    $0.01V = 0.01V_1 + 0.2\, v_2$ $\Rightarrow \frac{20}{100} = \frac{V_2}{V_1}$

    $V = V_1 + 20\, V_2$ $\Rightarrow V_1 = 5V_2$.

    $V = 5\, V_2 + 20\, V_2 = 25\, V_2$

    Now $R = u\sqrt{\frac{2h}{g}}$

    $20 = V_2\sqrt{\frac{2\times 5}{10}} \Rightarrow V_2 = 20$

    Now $V = 25 \times 20 = 500$ m/s.
23. **Ans.** (C)
24. **Ans.** (A)
25. **Ans.** (C)
26. **Ans.** (B) **Reason:** $V_e = \sqrt{2}\, V$

    $KE = \frac{1}{2} m (\sqrt{2}\, V)^2 = mv^2$
27. **Ans.** (A) **Reason:** No. of turn in element $dx = \frac{N}{b-a} . dx$

    $dB = \frac{\mu_0 I}{2x} . \frac{N}{b-a} dx$

    $B = \frac{\mu_0 IN}{2(b-a)} \int_a^b \frac{dx}{x} = \frac{\mu_0 IN}{2(b-a)} \log_e \frac{b}{a}$
28. **Ans.** (B) **Reason:** A sin ωt + A sin $\left(\omega t + \frac{2\pi}{3}\right) + B\sin(\omega t + \phi) = 0$

    $B \sin (\omega t + \phi) = \frac{A}{2} .2\sin (\omega t + \pi/3)$

    $\Rightarrow B = A, \phi = \pi/3$.
29. **Ans.** (ABD)
30. **Ans.** (AC)
31. **Ans.** (C, D)
32. **Ans.** (B, C)
33. **Ans.** 4 **Reason.** $Z^2 = R^2 + X_C^2$

    $(R\sqrt{1.25})^2 = R^2 + X_C^2$

    $X_C^2 = 0.25R^2$

    $\Rightarrow X_C = \frac{R}{2} = \frac{1}{500\ C}$

    $\Rightarrow R = 1/125C$

    Now $Z = RC = \frac{1}{125\ C} \times C = \frac{1}{125}$ sec = 8 milli sec.
34. **Ans.** 7 **Reason.** $KE = \frac{12400}{2000} - 4.7 = 1.5ev$

    $KE = 2v = q.\frac{kq}{r}$

    $1.5 \times e = \frac{9\times 10^9 \times n^2 e^2}{r}$

    $\Rightarrow 1.5 = \frac{9\times 10^9 \times n^2 \times 1.6\times 10^{-19}}{10^{-2}}$

    $\Rightarrow 1.5 = 9\times 10^9 \times n^2 \times 1.6 \times 10^{-17}$

    $n^2 = \frac{1}{9\times 10^{-8}} = \frac{10^8}{9} \Rightarrow n = \frac{10^4}{3} = 0.3\times 10^4 = 3\times 10^3$
35. **Ans.** 5 **Reason.** $T = \frac{2u\sin\theta}{g} = \sqrt{3}$ sec.

    $\frac{u^2 \sin 2\theta}{g} - \frac{1}{2}at^2 = 1.15 \Rightarrow a = 5\ m/s^2$.
36. **Ans.** 4 **Reason.** $1/2\ kx^2 + F\times x = 1/2\ mv^2$

    $1/2.2\ (0.0c)^2 + 1/10 \times 0.18\ g(0.06) = 1/2.\ 0.18\ v^2$

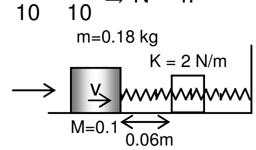
    $18 \times 2 \times 10^{-4} - 18 \times 10^{-4} = 1/2 \times 18 \times 10^{-2}\ v^2$

    $2 \times 10^{-2} + 6 \times 10^{-2} = v^2/2.$

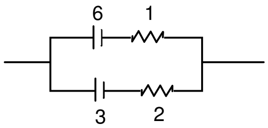
    $V^2 = (4 + 12) \times 10^{-2}$

    $V = \sqrt{16} \times 10^{-1} = 4 \times 10^{-1}$

    $\frac{4}{10} = \frac{N}{10} \Rightarrow N = 4.$



37. **Ans.** 5v **Reason.** $E = \dfrac{\frac{6}{1}+\frac{3}{2}}{\frac{1}{1}+\frac{1}{2}} = \dfrac{\frac{12+3}{2}}{3/2} = \dfrac{15}{3} = 5V.$



38. **Ans.** 0 **Reason.** $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$$\frac{7/4}{V} - \frac{1}{-18} = \frac{\frac{7}{4}-1}{6} = \frac{7}{4V} + \frac{1}{18} = \frac{3}{4\times 6}$$

$$\frac{7}{4V} = \frac{3}{24} - \frac{1}{18} = \frac{5}{72}$$

$$V = \frac{7\times 72}{4\times 5} = \frac{7\times 18}{5}\text{cm}$$

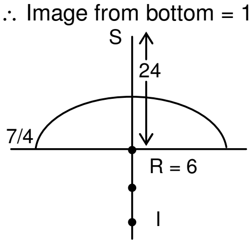
$$\frac{\mu_2}{v} = \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{4}{3v} - \frac{7/4}{\left(\frac{7\times 18}{5}\right)} = \frac{\frac{4}{3}-\frac{7}{4}}{\infty} = 0$$

$$\frac{4}{3V} = \frac{7}{4}\times\frac{5}{18\times 7}$$

$$V = \frac{4}{3}\times\frac{4\times 18}{5} = 18$$

$\therefore$ Image from bottom = 18 – 18 = 0.



39. **Ans.** A $\to$ p, r, t; B $\to$ p, r, C $\to$ q, s, D $\to$ r, t

40. **Ans.** A $\to$ p, t; B $\to$ p, s, C $\to$ q, s, D $\to$ q, r

41. **Ans**. (D) **Reason :** $\lim\limits_{x\to 0}[1 + x\ \ln\ (1+b^2)]^{1/x} = 2b\sin^2\theta$

$= 1 + b^2 = 2b\sin^2\theta$

$$\sin^2\theta = \left(\frac{1+b^2}{2b}\right) \le 1$$

$1 + b^2 \le 2b$
$b^2 – 2b + 1 \le 0$
$(b – 1)^2 \le 0$
$(b – 1)^2 = 0$
$b – 1 = 0$
$b = 1$
$\sin^2\theta = 1$
$\sin\theta = \pm 1$
$\theta = \pm\pi/2$

42. **Ans.** (C) **Reason :** $R_1 = \int_{-1}^{2} xf(x)dx$ as $f(x) = f(1 – x)$

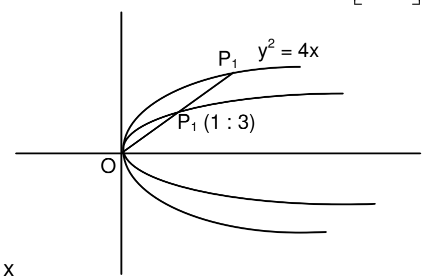
$= \dfrac{2+(-1)}{2}\int_{-1}^{2} f(x)dx$ given that $R_2 = \int_{-1}^{2} f(x)dx$

$R_1 = \dfrac{1}{2}(R_2)$, $R_2 = 2R_1$.

43. **Ans.** (A) **Reason :** $f(x) = x^2$, $g(x) = \sin x$

fogogof (x) = (gogof)x
$\{\sin(\sin x^2)\}^2 = \sin(\sin x^2)$
$t^2 = t$, t = 0, 1
$\sin(\sin x^2) = 0$
$\sin x^2 = 0$
$x^2 = n\pi$
$x = \pm\sqrt{n\pi}$
n = {0,1,2,.......}
$\sin(\sin x^2) = 1$
$\sin(x^2) = (4x+1)\pi/2$

44. **Ans.** (C) **Reason :** $p(at^2, 2at)$, $p(t^2, 2t)$ $p_1\left[\dfrac{t^2}{4},\dfrac{t}{2}\right]$,



$\dfrac{y^2}{x} = 1$, $y^2 = x$

45. **Ans.** (B) **Reason:** . $(6,3)\in \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1 \Rightarrow \dfrac{36}{a^2} - \dfrac{9}{b^2} = 1$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{dy}{dx} = -\frac{\left(-\frac{2y}{b^2}\right)}{\left(\frac{2x}{a^2}\right)} = \frac{a^2y}{b^2x}$$

$$m_N = -\frac{dx}{dy} = -\frac{b^2x}{a^2y}$$

$$\Rightarrow m_N\Big/_{y=3}^{x=6} = \frac{-b^2(6)}{a^2(3)} = \frac{-2b^2}{a^2}$$

Eqn. of normal: $y - 3 = \dfrac{-2b^2}{a^2}$ $(x – 6)$

$X = 9$, $y = 0$, $-3 = \dfrac{-2b^2}{a^2}$ (3)

46. **Ans.** (B) **Reason:**

$x^2 + bx – 1 = 0,$
$x^2 + x + b = 0$
\- \- \-
$x(b-a) – (b+1) = 0$ $\quad x = \dfrac{b+1}{b-1}$

X(x+1) + b = 0

$$\frac{b+1}{b-1}\left(\frac{2b}{b-1}\right) + 1 = 0$$

b$\neq$0, b$\neq$1, $\dfrac{2(b+1)}{(b-1)^2} + 1 = 0$

$(b – 1)^2 + 2(b+1) = 0$

(b – 1)2 + 2(b+1) = 0

b2 + 3 = 0

$b = \sqrt{-3} = \pm\sqrt{3}i$

$b = -\sqrt{3}\ i$

47. **Ans.** (A) **Reason:** 4 cases possible

A	B	C
ω	ω	ω
ω	ω	ω^2
ω	ω^2	ω
ω	ω^2	ω^2
ω^2	ω	ω
ω^2	ω	ω^2
ω^2	ω^2	ω
ω^2	ω^2	ω^2

48. **Ans.** (D) **Reason:** (r – 1)2 + 4 = r2

$4 = r^2 - (r-1)2$

4 = (2r – 1)

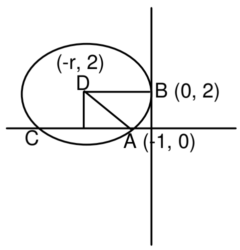
2r = 5

r = 5/2

C(-5/2, 0)

∴ D(-5/2, 0)

C[-4, 0]



49. **Ans.** (A, B, C, D) **Reason:** $f(x) = \begin{cases} -x-\frac{\pi}{2}, & x \le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x-1, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$

It 5 continuous at x = -π/2 = LHL= RHL = exact value

Not diff at x = 0 (IHD ≠ RHD, 0 ≠ 1)

f(x) is diff at x = 1, LHD = RHD, 1 = 1

f(x) is diff at $x = -\frac{3}{2}$ as $-\frac{3}{2} < -\frac{\pi}{2}$

$f'(x) = -1$

50. **Ans.** (C, D) **Reason:** f : (0, 1) → IR

$y = f(x) = \frac{b-x}{1-bx}$ b is a const 0 < b < 1

f is invertible:

If f(y) = x

$$\frac{b-y}{1-by} - \frac{b-\frac{b-x}{1-bx}}{1-b\left(\frac{b-x}{1-bx}\right)} = \frac{b-b^2x-b+x}{1-bx-b^2+bx} = \frac{x(1-b^2)}{(1-b^2)} = x$$

$y = \frac{b-x}{1-bx}$

y – bxy = b – x

x – bxy = b – y

x(1 – by) = b – y

$f^{-1}(y) = x = \frac{b-y}{1-by}$

$f^{-1}(x) = \frac{b-x}{1-bx} = f(x)$

f^{-1} is diff. (0, 1)

51. **Ans.** (A, B, D) **Reason:** $y^2 = 4x$

x = 1

Equation of tangent: $y = mx + \frac{1}{m}$

$P\left(\frac{1}{m^2}, \frac{2}{m}\right)$

Equation of normal : $y - \frac{2}{m} = -\frac{1}{m}\left(x - \frac{1}{m^2}\right)$

$(a, b) \in y - \frac{2}{m} = -\frac{1}{m}\left(x - \frac{1}{m^2}\right)$

$6 - \frac{2}{m} = -\frac{1}{m}\left(9 - \frac{1}{m^2}\right)$

$6 - \frac{2}{m} = -\frac{9}{m} + \frac{1}{m^3}$

$6 = \frac{1}{m^3} - \frac{7}{m}$

$6m^3 = 1 - 7x^2$

$6m^3 + 7x^2 - 1 = 0$

$(x + 1)(6m^2 + m - 1) = 0$

$m = -1,\ m = \frac{-1 \pm \sqrt{1+24}}{12}$

$m = \frac{-1 \pm 5}{12}$

$m = \frac{1}{3} \text{ or } -\frac{1}{2}$

P(9, 6), m_N = 1, -3, 2

y – 6 = (x – 9), y – 6 = -3(x – 9), y – 6) = 2(x – 9)

y – x + 3 = 0, y + 3x – 33 = 0, y – 2x + 12 = 0

52. **Ans.** (A,D) **Reason:** E and F are independent events

$P(E \cap F') + P(E' \cap F) = \frac{11}{25}$

$P(E' \cap F') = \frac{2}{25}$

Let P(E) = x

P(F) = y

$P(E)[1 - P(F)] + P(F)[1 - P(E)] = \frac{11}{25}$

$\Rightarrow x(1 - y) + y(1 - x) = \frac{11}{25}$

$\Rightarrow x + y - 2xy = \frac{11}{25}$ ….(1)

$(1 - x)(1 - y) = \frac{2}{25}$

$1 - x - y + xy = \frac{2}{25}$

$1 - (x + y) + xy = \frac{2}{25}$ ….(2)

Equation (1) + 2 × equation (2)

x + y – 2xy

$2 - 2x - 2y + 2xy = \frac{11}{25} + \frac{4}{25}$

$$2 - x - y = \frac{15}{25} = \frac{3}{5}$$

$$x + y = 2 - \frac{3}{5} = \frac{7}{5} \quad ....(3)$$

$$xy = \frac{2}{25} - 1 + \frac{7}{5} = \frac{2-25+35}{25}$$

$$xy = \frac{12}{25} \quad ......(4)$$

$$x = \frac{3}{5} \text{ or } \frac{4}{5}$$

y = 4/5 or 3/5

53. **Ans.** 9 **Reason:** $\vec{a} = -\hat{i} - \hat{k}$

$\vec{b} = -\hat{i} + \hat{j}$

$\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

Let $\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}$

$\vec{r}\,.\,\vec{a} = 0 \Rightarrow -r_x - r_z = 0$

$r_x + r_z = 0$

Let $r_x = r$, $r_z = -r$

$\vec{r} = r\hat{i} + r_y\hat{j} - r\hat{k}$

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r & r_y & -r \\ -1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{vmatrix}$$

$\hat{i}(r) - \hat{j}(-r) + \hat{k}(r + r_y) = -3\hat{i} - \hat{j}(3) + \hat{k}(3)$

$\begin{bmatrix} r = -3 \\ r_y = 6 \end{bmatrix} \vec{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$

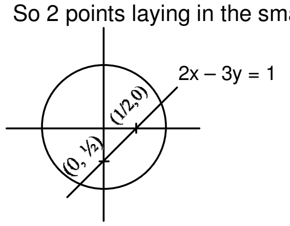
$\vec{r}\,.\,\vec{b} = 3 + 6 + 0 = 9.$

54. **Ans.** 2 **Reason.** Number of points below 2x – 3y = 1 is $2x - 3y - 1 \geq 0$

$\left(2, \frac{3}{4}\right) \Rightarrow -\frac{5}{4}$, $\left(\frac{5}{2}, \frac{3}{4}\right) \Rightarrow \frac{7}{4}$, $\left(\frac{1}{4}, -\frac{1}{4}\right) \Rightarrow \frac{1}{4}$,

$\left(\frac{1}{8}, \frac{1}{4}\right) \Rightarrow -\frac{3}{2}$

So 2 points laying in the smaller part



55. **Ans.** 3

56. **Ans.** 2 **Reason.** $x^4 - 4x^3 + 12x^2 + x - 1 = 0$

Number of positive real roots = 2

$x \to -x$, $x^4 + 4x^3 + 12x^2 - x - 1 = 0$

Number of Negative real roots = 1

Number of imaginary roots = 4 – (2 +1) = 1 which is wrong

As imaginary roots occur in (pairs)

Number of distinct real roots = 4 – 2 = 2.

57. **Ans.** 0 **Reason:** Let g(x) = x(x – 2)

g(0) = 0

g(2) = 0

y(0) = 0 $\quad$ g'(x) = 2x – 2

$$\left.\begin{matrix} x = 0 \\ y = 0 \end{matrix}\right\} \frac{dy}{dx} + y + 2x - 2 = 2(x^2 - 2x)(x - 1)$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=0 \\ y=0}} + 0 + 0 - 2 = 0$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=0 \\ y=0}} = (2)$$

y = 2x

$y(2) = 2 \times 2 = 4$

58. **Ans.** 8 **Reason:** Let $M = \begin{bmatrix} a & b & c \\ p & q & r \\ l & m & n \end{bmatrix}$

a + q + n = ? $\qquad M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

b = –1, q = 2, m = 3

$$M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\left.\begin{matrix} a - b = 1 \\ p - q = 1 \\ l - m = 0 \end{matrix}\right\} \begin{matrix} a = 1 + b = 0 \Rightarrow a = 0 \\ p = 1 + q = 3 \Rightarrow p = 3 \\ l = m = 3 \Rightarrow l = 3 \end{matrix}$$

$$M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$a + b + c = 0 \Rightarrow 0 + (-1) + c = 0$

c = –1

$p + q + r = 0 \Rightarrow r = -(p + q) = -5$

r = -5

$l + m + n = 12 \Rightarrow n = 6$

Now, a + q + n = 0 + 2 + 6 = 8

59. **Ans.** (A) → (r), (B) → (p), (C) → (s), (D) → (p, q, r, s)

60. **Ans.** (A) → (s), (B) → (p, t), (C) → (r), (D) → (r)