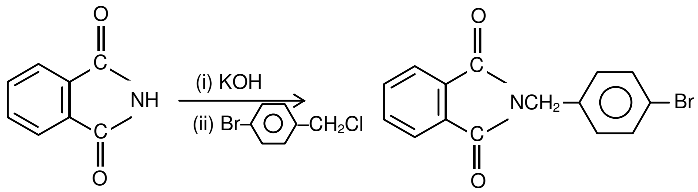
IIT JEE SOLUTION PAPER-1

SOLUTIONS



1. **Ans. (A) Reason:**
2. **Ans. (D) Reason:** $Ba(N_3)_2 \longrightarrow Ba + N_2$.
3. **Ans.** (B) **Reason:** $[NiCl_4]^{2-} \rightarrow$ Tetrahedra

   $[Ni(CN)_4]^{2-} \longrightarrow$ Square planar

   $[Ni(H_2O)_6]^{+2}$ = octahedral.
4. **Ans.** (A) **Reason:** …..
5. **Ans.** (C) **Reason:** $M = \frac{W}{M} \times \frac{1000}{V} = \frac{120}{60} \times \frac{1000}{\frac{1120}{1.15}} = 2.05$
6. **Ans.** (D) **Reason:** …..
7. **Ans.** (C) **Reason:** Lower is the pK_a, stronger will be the acid.
8. **Ans.** (A), (B), (C) **Reason:** …..
9. **Ans.** (A, B, C) **Reason:** …..
10. **Ans.** (A), (D) **Reason:** …..
11. **Ans.** (A), (C), (D) **Reason:** …..
12. **Ans.** (B) **Reason:** …..
13. **Ans.** (A) **Reason:** …..
14. **Ans.** (C) **Reason:** …..
15. **Ans.** (D) **Reason:** …..
16. **Ans.** (B) **Reason:** …..
17. **Ans.** (5) **Reason:** …..
18. **Ans.** 5 **Reason:** …..
19. **Ans.** 4 **Reason:** …..
20. **Ans.** 5 **Reason:** …..
21. **Ans.** 7 **Reason:** …..
22. **Ans.** 9 **Reason:** …..
23. **Ans.** 5 **Reason:** …..
24. **Ans.** (D) **Reason:** …..
25. **Ans.** (A) **Reason.** $\eta' = n\left(\frac{V+x}{V-x}\right) = 8\left(\frac{330}{310}\right) = 8.5kHZ$
26. **Ans.** (A) **Reason. Ans.** $dw = \frac{nRdT}{r-1}$

   $T_1V_1^{r-1} = T_2V_2^{r-1}$

   $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{r-1} = \left(\frac{5.6}{0.7}\right)^{\frac{5}{3}-1} = (8)^{2/3} = 4$

   $\Rightarrow T_2 = 4T_1$

   5.6 lit helium at STP = 1/4 mole

$dw = \frac{nRdT}{r-1} = \frac{\frac{1}{4}\times R\times 3T}{5/3-1} = \frac{3RT/4}{2/3} = \frac{9RT}{8}$

27. **Ans.** (A) **Reason.** $\vec{A}(-a\hat{j})\times(a\hat{i}+a\hat{k})$

= $a^2(\hat{k})-a^2(\hat{i})$

$\phi = \vec{E}.\vec{A} = E_0(\hat{i}).\{a^2(\hat{k})-a^2(\hat{i})\} = E_0a^2$.

28. **Ans.** (A) **Reason. Ans.** $\frac{\lambda}{6561} = R\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{5R}{36}$

$\frac{1}{\lambda} = R\times 4\left(\frac{1}{4}-\frac{1}{16}\right) = \frac{3R}{4}$

$\lambda = 6561 \times 5/27 = 1215$ Å

29. **Ans.** (D) **Reason.** $T = mL\omega^2$

$\omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324}{0.5\times 0.5}} = 36$.

30. **Ans.** (B) **Reason.** $\frac{x}{10} = \frac{52+1}{48+2} = \frac{53}{50}$

$x = 53/5 = 10.6$ ohm.

31. **Ans.** (C) **Reason. Ans.** Heat flows remains constant in series

$K(\Delta\theta)$ = constant

$\Rightarrow$ For E, 6k > 2k

$\Rightarrow \Delta\theta$ for E is less.

32. **Ans.** (A, D) **Reason:** …..
33. **Ans.** (B), (D) **Reason.** …
34. **Ans.** (A), (b), (d). **Reason.** …
35. **Ans.** (D) **Reason:** …..
36. **Ans.** (C) **Reason:** …..
37. **Ans.** (B) **Reason:** …..
38. **Ans.**(C) **Reason:** …..
39. **Ans.** (B) **Reason:** $\omega = \sqrt{\frac{Ne^2}{m\epsilon_0}}$

$= \sqrt{\frac{4\times 10^{27}\times(1.6\times 10^{-19})^2}{10^{-30}\times 10^{-11}}}$

$\Rightarrow \omega = 3.2 \times 10^{15}$

$\Rightarrow f = \frac{3.2\times 10^{15}}{2\times 3.14} = 0.5 \times 10^{15}$

$C = \lambda f$

$\lambda = 3\times 10^8 / 0.5 \times 10^{15} = 6 \times 10^{-7} = 600$ nm

40. **Ans.** 9 **Reason:** …..
41. **Ans.** 4 **Reason:** …..
42. **Ans.** 6**Reason:** …..
43. **Ans.** 5 **Reason:** $mg \sin\theta + \mu mg \cos\theta = (mg \sin\theta - \mu mg \cos\theta)3$

$2\, mg \sin\theta = 4\,\mu\, mg \cos\theta$

$2\mu = 1$

$\mu = 1/2$

$N = 10\,\mu = 10\left(\frac{1}{2}\right) = 5$

44. **Ans.** 1 **Reason:** $\frac{dN}{dt} = \lambda N = 10^{10}$

$= \frac{N}{\tau} = 10^{10}$

$\Rightarrow N = 10^{10} \times 10^{9} = 10^{19}$

mass of 1 atom = 10^{-25} kg

mass of 10^{19} atom = $10^{-25} \times 10^{19} = 10^{-6}$ kg

$= 10^{-3}$ gm = 1 mg.

45. **Ans.** 3 **Reason:** $2\sqrt{2}\,ra = K\frac{q^2}{(\sqrt{2}a)^2}$

$\Rightarrow a^3 = \frac{Kq^2}{4\sqrt{2}r}$

$\Rightarrow a = K\left(\frac{q^2}{r}\right)^{1/3}$

$\Rightarrow N = 3$

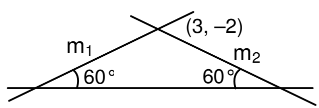
46. **Ans.** 3 **Reason:** $mg = YA \propto \Delta\theta$

$mg = 10^{11} \times \pi \times 10^{-6} \times 10^{-5} \times 10$

$\Rightarrow m = \pi$ kg = 3.14 kg

$\Rightarrow m = 3$ kg (nearly)

47. **Ans.** $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$ **Reason:**



$\sqrt{3}x + y = 1\ (-\sqrt{3})$

m_1 or $m_2 = \frac{-\sqrt{3} \pm \tan 60}{1 \mp (-\sqrt{3})\tan 60}$

$= \frac{-\sqrt{3} \pm \sqrt{3}}{1 \mp (-\sqrt{3})(\sqrt{3})} = \frac{-\sqrt{3} \pm \sqrt{3}}{1 \mp (-3)}$

$= \frac{0}{4}$ or $\frac{-2\sqrt{3}}{-2}$

$= 0$ or $\sqrt{3}$

equation of lines :

$y + 2 = 0$

$y + 2 = \sqrt{3}(x - 3)$

$y + 2 = \sqrt{3}x - 3\sqrt{3}$

$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

48. **Ans.** $\frac{1}{2}$ **Reason:** $(2x)^{\ln 2} = (3y)^{\ln 3}$

$(3)^{\ln x} = (2)^{\ln y}$

This is possible if $2x = 1$ and $3y = 1$

$x = \frac{1}{2}$ and $y = \frac{1}{3}$

49. **Ans.** $\frac{1}{4}\ln(3/2)$ **Reason:** $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin(x^2)dx}{\sin x^2 + \sin(\ln 6 - x^2)}$

Let $t = x^2$

$dt = 2x\,dx$

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = \frac{b-a}{2}$

$= \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t\, dt}{\sin t + \sin(\ln 6 - t)}$

$= \frac{1}{2}\frac{\ln(3/2)}{2} = \frac{1}{4}\ln(3/2)$

50. **Ans.** (C) **Reason.** Let $\vec{v}$ is vector in the plane of $\vec{a}$ & $\vec{b}$

$\vec{v} = l\,\vec{a} + m\,\vec{b}$

$= l(\hat{i}+\hat{j}+\hat{k}) + m(\hat{i}-\hat{j}+\hat{k})$

$= (l + m)\,\hat{i} + (l - m)\,\hat{j} + (l + m)\,\hat{k}$

Projection of $\vec{v}$ on $\vec{c}$ is $\frac{1}{\sqrt{3}}$

$\frac{\vec{v}\,.\,\vec{c}}{|\,\vec{c}\,|} = \frac{1}{\sqrt{3}}$ $\qquad$ $\vec{c} = \hat{i} - \hat{r} - \hat{k}$

$\frac{(l+m) - (l-m) - (l+m)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$l - m = -1$	$L = m\ -1$

l	1	0	2	–2
m	2	1	3	–1

$1(\hat{i}+\hat{j}+\hat{k})$
$2(\hat{i}-\hat{j}+\hat{k})$

$3\hat{i} - \hat{j} + 3\hat{k}$

51. **Ans.** P = Q. **Reason:** P = {sinθ – cosθ = √2 cosθ}

Q = {sinθ + cosθ = √2sinθ}

sinθ = (√2 + 1) cosθ

tanθ = √2 + 1

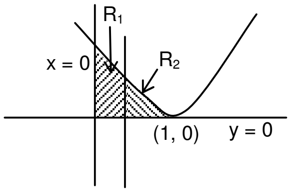
cosθ = sinθ (√2 – 1)

cotθ = √2 – 1

$\tan\theta = \frac{1}{\sqrt{2}-1} = \sqrt{2} + 1$

∴ P = Q.

52. **Ans.** $\frac{1}{2}$ **Reason:** $y = (1 - x)^2$



$R_1 - R_2 = \frac{1}{4}$

$R_1 = \int_0^b (x-1)^2 dx = \left[\frac{(x-1)^3}{3}\right]_0^b$

$= \frac{(b-1)^3}{3} + \frac{1}{3}$

$R_2 = \int_b^1 (x-1)^2 dx = \left[\frac{(x-1)^3}{3}\right]_b^1$

$0 - \frac{(b-1)^3}{3}$

$R_1 - R_2 = \frac{1}{4}$

$\frac{(b-1)^3}{3} + \frac{1}{3} + \frac{(b-1)^3}{3} = \frac{1}{4}$

$\frac{2(b-1)^3}{3} = \frac{1}{4} - \frac{1}{3} = \frac{-1}{12}$

$(b-1)^3 = -\frac{1}{12} \times \frac{3}{2} = -\frac{1}{8}$

$b - 1 = -\frac{1}{2}$

$b = 1 - \frac{1}{2} = \frac{1}{2}$

53. **Ans.** (C) **Reason.** $\alpha, \beta \in x^2 - 6x - 2 = 0$ $\alpha > \beta$

α or $\beta = \frac{6 \pm \sqrt{36+8}}{2} = \frac{6 \pm 2\sqrt{11}}{2}$

$= 3 \pm 2\sqrt{11}$.

$\alpha = 3 + \sqrt{11}$, β $3 - \sqrt{11}$

$q_n = (\alpha)^n - \beta^n$, $n \geq 1$

$q_n = (3 + \sqrt{11})^n - (3 - \sqrt{11})^n$

$\frac{q_{n+1} - 2q_{n-1}}{2q_n}$

$= \frac{(3+\sqrt{11})^{n+1} - (3-\sqrt{11})^{n+1} - 2(3+\sqrt{11})^{n-1} + 2(3-\sqrt{11})^{n-1}}{2q_n}$

$= \frac{(3+\sqrt{11})^{n-1}\left[(3+\sqrt{11})^2 - 2\right] - (3-\sqrt{11})^{n-1}\left[(3-\sqrt{11})^2 - 2\right]}{2q_n}$

$= \frac{6(3+\sqrt{11})^{n-1}\left[3+\sqrt{11}\right] - 6(3-\sqrt{11})^{n-1}\left[3-\sqrt{11}\right]}{2q_n}$

$= \frac{6}{2}\left[\frac{(3+\sqrt{11})^n - (3-\sqrt{11})^n}{q_n}\right]$

$\therefore \dfrac{q_{n+1} - 2q_{n-1}}{2q_n} = 3$

for n = 9, $\dfrac{q_{10} - 2q_8}{2q_9} = 3$.

54. **Ans.** (B), (D) **Reason:** Hyperbola : $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$e = \dfrac{\sqrt{a^2 + b^2}}{a}$

Ellipse : $x^2 + 4y^2 = 4$

$\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$

$e' = \dfrac{\sqrt{4-1}}{2} = \dfrac{\sqrt{3}}{2}$

Given that

$e \times e' = 1$

$\dfrac{\sqrt{a^2 + b^2}}{a} \times \dfrac{\sqrt{3}}{2} = 1$

$\dfrac{\sqrt{a^2 + b^2}}{a} = \dfrac{2}{\sqrt{3}}$

Squaring both side we get, $3(a^2 + b^2) = 4a^2$

$\Rightarrow 3b^2 = a^2$

Focus of the ellipse = $\left[\mp\sqrt{3}, 0\right]$.

Hyperbola passes through focus of ellipse

$\Rightarrow \dfrac{3}{a^2} - \dfrac{0}{b^2} = 1$

$\Rightarrow \dfrac{3}{a^2} = 1 \Rightarrow a^2 = 3$

Now $3b^2 = a^2 = 3$

$\therefore b^2 = 1$

The equation of hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\dfrac{x^2}{3} - \dfrac{y^2}{1} = 1$

$x^2 - 3y^2 = 3$ (option (D))

$e_{Hyp} = \dfrac{\sqrt{a^2 + b^2}}{a} = \dfrac{\sqrt{3+1}}{\sqrt{3}} = \dfrac{\sqrt{4}}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

$f_{Hyp} = \left[\pm\sqrt{a^2 + b^2}, 0\right]$ = (± 2, 0). (option (B)).

55. **Ans.** (B), (C) **Reason:** $f : R \to R$ be a function such that

$f(x + y) = f(x) + f(y) \ \forall \ x, y \in R$

f(x) = K (x), f ' (x) = K (constant)

f(x) is differentiable at x = 0

f(x) is continuous, $\forall \ x \in R$ (B)

f ' (x) is constant $\forall \ x \in R$ (C).

56. **Ans.** (C) **Reason:** M, N are two (3×3) skew symmetric matrices.

$M^T = -M$, $N^T = -N$, $MN = NM$

$(MN)^T$

$= N^T M^T$

$= (-N)(-M)$

$= NM$

$= MN$

i.e. MN is symmetric Matrix

$M^2N^2 (M^T N)^{-1} (MN^{-1})^T$

$= M^2N^2 . N^{-1} (M^T)^{-1} (N^{-1})^T M^T$

$= M^2N . (-M)^{-1} (-N)^{-1} (-M)$

$= -M^3N (NM)^{-1}$

$= -M^2 (MN)(MN)^{-1}$

$= -M^2 I$

$= -M^2$

Again : $(M N)^2 (-MN)^{-1}(MN^{-1})^T$

$= (MN)^2 (-MN)^{-1} (N^T)^{-1} (M)^T$

$= (MN)^2 (-MN)^{-1} (-N)^{-1} (-M)$

$= (MN)^2 (MN)^{-1} (N)^{-1} (-M)$

$= MN . N^{-1} (-M)$

$= -M^2$.

57. **Ans.** (A), (D) **Reason:** Let $\hat{i}+\hat{j}+2\hat{k}$, $\hat{i}+2\hat{j}+\hat{k}$ and $x\hat{i}+y\hat{j}+z\hat{k}$ are coplaner

$$\Rightarrow \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ x & y & z \end{vmatrix} = 0$$

$\Rightarrow (2z - y) - (z - x) + 2(y - 2x) = 0$

$\Rightarrow -3x + y + z = 0$

$\Rightarrow 3x - y - z = 0$ …………..(1)

also $(x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+\hat{j}+\hat{k}) = 0$

$\Rightarrow x + y + z = 0$ …………. (2)

from (1) and (2)

$x = 0$, $y + z = 0$

$$(\hat{j}-\hat{k}) = \begin{array}{l} y=1 \\ \underline{z=-1} \\ y+z=0 \end{array}$$

$$(-\hat{j}+\hat{k}) = \begin{array}{l} y=-1 \\ \underline{z=-1} \\ y+z=0 \end{array}$$

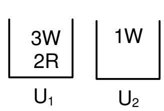
58. **Ans.** (D) **Reason:** …..

59. **Ans.** (A) **Reason:** …..

60. **Ans.** (B) **Reason:** …..

61. **Ans.**(B) **Reason:** …..

62. **Ans.** (D) **Reason:**



H $\Rightarrow$ 1 ball

T $\Rightarrow$ 2 balls

$$\left.\begin{aligned} P(H\,W_1\,W_2) &= \frac{1}{2}\times\frac{3}{5}\times 1 \\ P(H\,R_1\,W_2) &= \frac{1}{2}\times\frac{2}{5}\times\frac{1}{2} \end{aligned}\right\}$$

$$P(T, 2W, W_2) = \frac{1}{2}\times\frac{3}{10}\times 1$$

$$P(T, 2R, W_2) = \frac{1}{2}\times\frac{1}{10}\times\frac{1}{3}$$

$$P(T, RW, W_2) = \frac{1}{2}\times\frac{6}{10}\times\frac{2}{3}$$

$$P\left(\frac{H}{W_2}\right) = \frac{P(H\cap W_2)}{P(W_2)}$$

$$= \frac{\left(\frac{3}{10}+\frac{1}{10}\right)}{\frac{3}{5}+\frac{1}{5}+\frac{3}{10}+\frac{1}{30}+\frac{12}{30}}$$

$$= \frac{\frac{2}{5}}{\frac{1}{2}\left(\frac{18+6+9+1+12}{30}\right)} = \frac{\frac{2}{5}}{\frac{23}{30}} = \frac{12}{23}$$

63. **Ans.** 1 **Reason:** …..

64. **Ans.** 2 **Reason:** …..

65. **Ans.** 8 **Reason:** …..

66. **Ans.** 9 **Reason:** …..

67. **Ans.**5 **Reason:** …..

68. **Ans.**7. **Reason:** Let $\theta = \frac{\pi}{n}$

$$\Rightarrow \frac{1}{\sin\theta} = \frac{1}{\sin 2\theta} + \frac{1}{\sin 3\theta}$$

$$\Rightarrow \frac{1}{\sin\theta} - \frac{1}{\sin 3\theta} = \frac{1}{\sin 2\theta}$$

$$\Rightarrow \frac{\sin 3\theta - \sin\theta}{\sin\theta \sin 3\theta} = \frac{1}{\sin 2\theta}$$

$$\Rightarrow \frac{2\cos 2\theta \cancel{\sin\theta}}{\cancel{\sin\theta}\sin 3\theta} = \frac{1}{\sin 2\theta}$$

$\Rightarrow$ $2\sin 2\theta \cos 2\theta = \sin 3\theta$

$\Rightarrow$ $\sin 4\theta = \sin 3\theta = \sin(\pi - 3\theta)$

$\Rightarrow$ $4\theta + 3\theta = \pi$

$\Rightarrow$ $7\theta = \pi$

$\Rightarrow$ $\theta = \frac{\pi}{7} = \frac{\pi}{n}$

So n = 7.

69. **Ans.** 6 **Reason:** $f : [1, \infty) \longrightarrow [2, \infty)$

$f(1) = 2$

$$6\int_1^x f(t)dt = 3x\,f(x) - x^3$$

$x \geq 1$

f(2) = _____

Differentiating we get,

$6f(x) = 3f(x) + 3xf\,'(x) - 3x^2$

$3f(x) = 3xf'(x) - 3x^2$

$f(x) = xf'(x) - x^2$

$$\frac{y}{x} = \frac{x}{x}\frac{dy}{dx} - \frac{x^2}{x}$$

$$\frac{y}{x} = \frac{dy}{dx} - x$$

$$\frac{dy}{dx} + y\left(-\frac{1}{x}\right) = x$$

$$\text{If } e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

$$y\left(\frac{1}{x}\right) = \int x.\frac{1}{x} + C$$

$$\frac{y}{x} = x + C$$

$f(x) = y = x(x + C)$

$f(1) = 2 \Rightarrow f(1) = 1(1 + C) = 2$

$\Rightarrow C = 1$

$f(x) = x(x + 1)$

$f(2) = 2(2 + 1) = 6.$